

HOMEWORK SET 06: THETA AND R EQUATIONS

Due Friday, February 7, 2025

Problem adapted from TZDII¹

8.30) a) Write down the complete θ equation for the case $\ell = 2, m = 1$

$$\frac{1}{\sin\theta} \frac{d}{d\theta} \left(\sin\theta \frac{d\Theta_{\ell,m}}{d\theta} \right) + \left(\ell(\ell+1) - \frac{m^2}{\sin^2\theta} \right) \Theta_{\ell,m} = 0 \quad \text{TZDII (8.65)}$$

b) Show that the $\Theta_{2,1}(\theta)$ solution below does satisfy this equation (TAKE THE DERIVATIVE OF $\Theta_{\ell,m}(\theta)$, SUBSTITUTE AND TAKE THE DERIVATIVE OF $\sin(\theta)$ TIME THIS AND DO THE ALGEBRA TO GET 0 = 0)

	$\ell = 0$	$\ell = 1$	$\ell = 2$
$m = 0$	$\Theta_{0,0} = \sqrt{\frac{1}{4\pi}}$	$\Theta_{1,0} = \sqrt{\frac{3}{4\pi}} \cos\theta$	$\Theta_{2,0} = \sqrt{\frac{5}{16\pi}} (3\cos^2\theta - 1)$
$m = \pm 1$		$\Theta_{1,\pm 1} = \mp \sqrt{\frac{3}{8\pi}} \sin\theta$	$\Theta_{2,\pm 1} = \mp \sqrt{\frac{15}{8\pi}} \sin\theta \cos\theta$
$m = \pm 2$			$\Theta_{2,\pm 2} = \sqrt{\frac{15}{32\pi}} \sin^2\theta$

8.39) a) Verify that

$$\frac{d}{dr^2} (rR) = \frac{2m_e}{\hbar^2} \left(-\frac{ke^2}{r} + \frac{E_R}{n^2} \right) (rR) \quad \text{TZDII (8.79)}$$

can be written as

$$\frac{d}{dr^2} (rR) = \left(\frac{1}{n^2 a_B^2} - \frac{2}{a_B r} \right) (rR) \quad \text{TZDII (8.107)}$$

Where $a_B = \frac{\hbar^2}{m_e k e^2}$ and $E_R = \frac{ke^2}{2a_B}$.

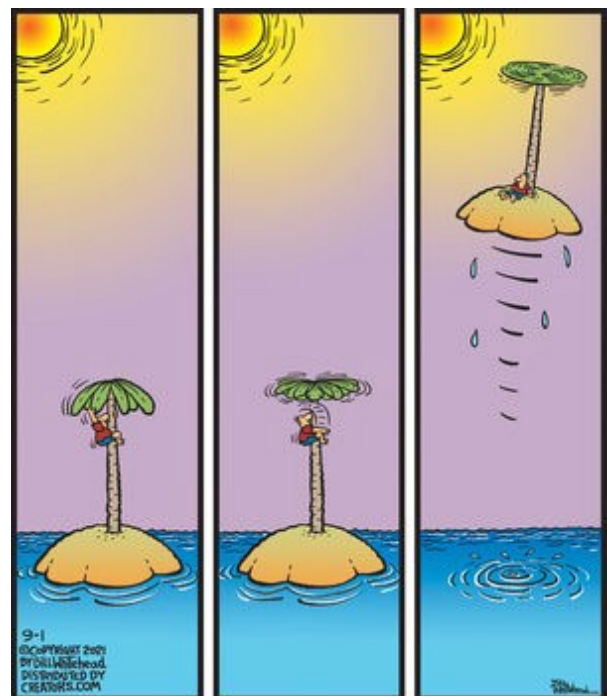
b) for the case that $n = 1$, prove that $R_{1s}(r) = e^{-r/a_B}$ is a solution of (8.107).

8.44) a) Verify that

$$R_{2,0}(r) = A \left(2 - \frac{r}{a_B} \right) e^{-r/2a_B}$$

Satisfies the radial Schrödinger Equation (8.107)

b) Find A from normalization.



¹ Taylor, Zafiratos, & Dubson, *Modern Physics for Scientists and Engineers*, 2nd Edition, Pearson, Prentice Hall, 2004