

HOMEWORK SET 19: CLASSICAL ENERGY DISTRIBUTION

Due Monday, March 31, 2025

PROBLEMS FROM OR AFTER THORNTON & REX (TRES 3RD ED.)¹

9.12 Altered) Consider the ideal gas H₂ at T = 293K.

a) Find v_{mp} in m/s

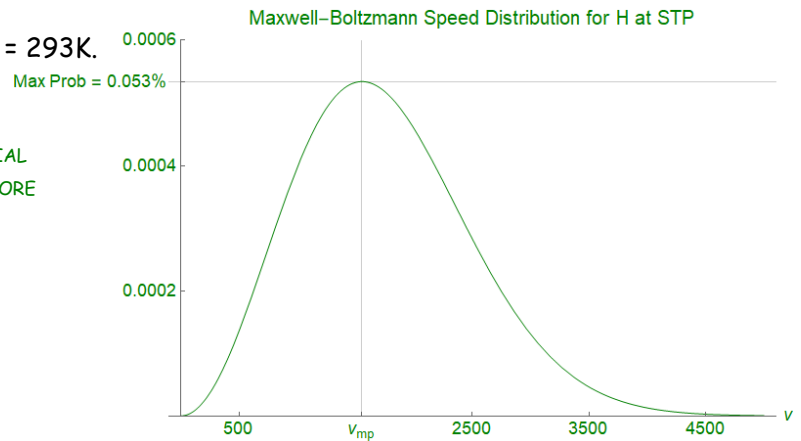
USE THE APPROXIMATION GIVEN IN EXAMPLE 9.4 FOR THE GIVEN RANGES. CALCULATE THE 3/2 AND EXPONENTIAL FACTORS OF EACH SEPARATELY AS I DID IN CLASS ... EXPLORE THE NUMBERS, DON'T JUST CALCULATE THEM!

b) 0.95 v_{mp} and 1.05 v_{mp}

c) 0.5 v_{mp} and 0.6 v_{mp}

d) 1.5 v_{mp} and 1.6 v_{mp}

COMMENT ON THE FRACTIONS OF MOLECULES IN EACH RANGE IN VIEW OF THE PLOT.



9.13 Altered) We found the r.m.s velocities of H₂ and N₂ gases at 14 °C = 287 K and compared them to Earth's escape velocity. Do this now for O₂ and CO₂ (WRITE OUT HOW THE ESCAPE VELOCITY IS DERIVED AND CALCULATE IT FOR YOURSELF).

9.15) Use the Maxwell-Boltzmann Energy Distribution Equation (9.26) to

a) Find the mean translational kinetic energy of an ideal gas and

b) compare your results with $\frac{1}{2}m\bar{v}^2$ and $\frac{1}{2}m\bar{v}^2$.

Hints: The M-B Energy Distribution is

$$F_{MB}(E) = \frac{8\pi}{\sqrt{2m^3}} \left(\frac{m}{2\pi kT} \right)^{3/2} e^{-E/kT} \sqrt{E} \quad (9.26)$$

The mean energy is then

$$\bar{E} = \int_0^{\infty} EF(E)dE$$

The CRC gives the integral (#369 as) as

$$369. \int_0^{\infty} x^n e^{-ax} dx = \begin{cases} \frac{\Gamma(n+1)}{a^{n+1}}, & (n > -1, a > 0) \\ \text{or} \\ \frac{n!}{a^{n+1}}, & (a > 0, n \text{ positive integer}) \end{cases}$$

9.2 Gamma Function (Generalized Factorial Function)

The gamma function, denoted $\Gamma(x)$, is defined by

$$\Gamma(x) = \int_0^{\infty} e^{-t} t^{x-1} dt, \quad x > 0$$

• Properties

$$\Gamma(x+1) = x\Gamma(x), \quad x > 0$$

$$\Gamma(1) = 1$$

$$\Gamma(n+1) = n\Gamma(n) = n! \quad (n = 1, 2, 3, \dots)$$

$$\Gamma(x)\Gamma(1-x) = \pi / \sin \pi x$$

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

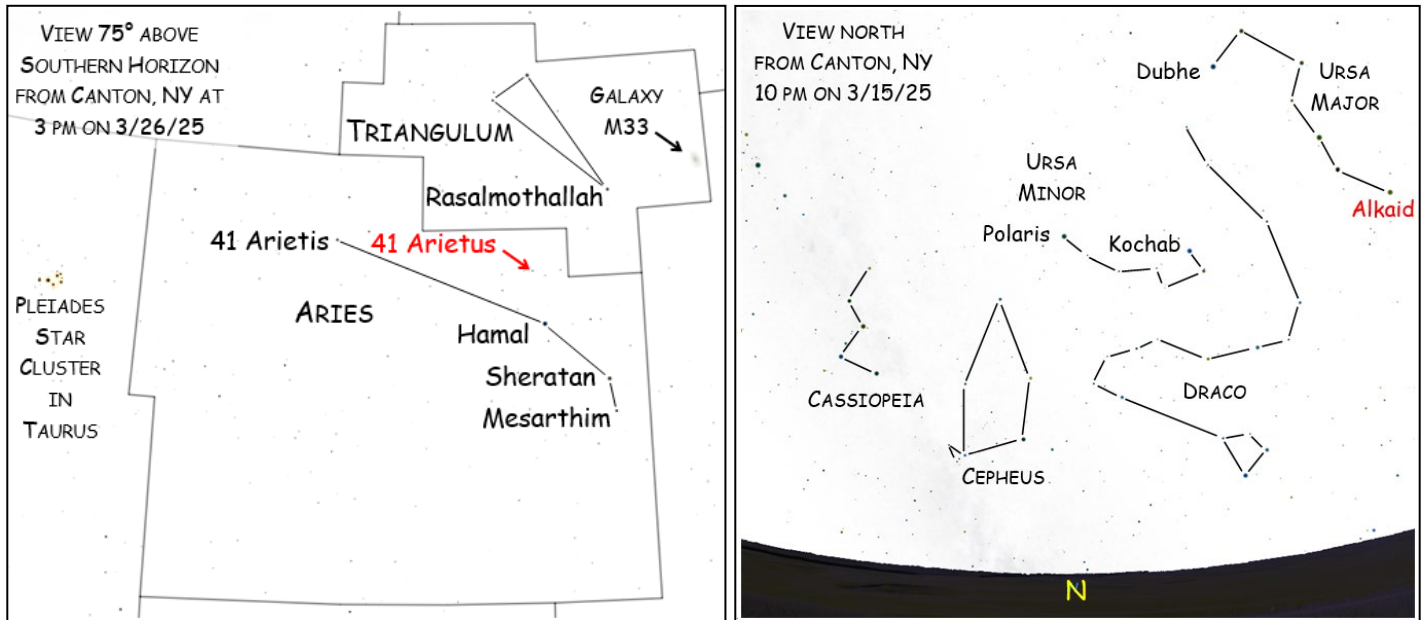
$$2^{2x-1}\Gamma(x)\Gamma\left(x + \frac{1}{2}\right) = \sqrt{\pi}\Gamma(2x)$$

Where the top form applies to your integral (with $E^{3/2}$). The Gamma (Γ) functions are explained on CRC pp. 128-129 [for $\Gamma(x)$ instead of $\Gamma(n)$]. The properties that apply here are circled in green on the image above ignore the integral in the explanation unless it interests you. You should get the integral you have to solve (not multiplied by any of the constants) to be the value to the right:

$$(kT)^{5/2} \left(\frac{3\sqrt{\pi}}{4} \right)$$

¹ Thornton & Rex, Modern Physics for Scientists and Engineers, 2nd Edition, Saunders, Harcourt Brace College Publishers, 2000

9.17) Near the surface of the star 14 Arietis, approximately 1 H atom per 10 million is in the first excited level ($n = 2$). Assume that the other atoms are in the ground state ($n = 1$). Estimate the temperature assuming that Maxwell-Boltzmann statistics are valid. (Hint: the density of states depends on the number of possible quantum states available in each n level) Answer = 6,761K



PROBLEM FROM SERWAY, MOSES & MOYER, MODERN PHYSICS (2nd ED.)²

1) a) Find the populations of the first and second excited states relative to the ground state for atomic hydrogen at 300K assuming that it obeys Maxwell-Boltzmann statistics.

b) Assuming Maxwell-Boltzmann statistics, find the populations of the first and second excited states relative to the ground state for atomic hydrogen heated to 15,500 K in Alkaid (η UMa, "Eta Ursae Majoris", the star at the end of the handle of the Big Dipper).

c) for the populations in b, given that the Balmer absorption lines are upward transitions from the first excited state and the Paschen absorption lines are upward transitions from the second excited state, which absorption lines should be stronger, Balmer or Paschen (for equal numbers of incoming photons with appropriate energies)? HINT: PARTICLES ARE MORE LIKELY TO MOVE FROM A MORE-POPULATED STATE TO A LESS-POPULATED STATE THAN VISE VERSA.)

Answers: a) $\frac{n_2}{n_1} \Big|_{300K} = 4e^{-394} \approx 0, \frac{n_3}{n_1} \Big|_{300K} = 9e^{-468} \approx 0$ b) $\frac{n_2}{n_1} \Big|_{15,000K} = 4e^{-7.63} = 0.0019, \frac{n_3}{n_1} \Big|_{15,000K} = 9e^{-9.06} = 0.001$

Red and Rover by Brian Basset



² Serway, Moses, & Moyer, Modern Physics, 2nd Edition, Saunders, Harcourt Brace College Publishers, 1997