7.15) USE THE IDENTITY $e^{i(\theta+\phi)} = e^{i\theta} e^{i\phi}$ TO PROVE THE TRIG IDENTITIES FOR $\cos(\theta+\phi)$ AND $\sin(\theta+\phi)$.

THE IDENTITIES TO DERIVE ARE

$$\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$$

$$\sin(\theta+\phi) = \sin\theta\cos\phi + \sin\phi\cos\theta$$

START WITH THE GIVEN IDENTITY.

$$e^{i(\theta+\phi)} = e^{i\theta} e^{i\phi}$$

APPLY EULER'S RELATION THAT $e^{i\theta} = \cos\theta + i\sin\theta$ TO BOTH SIDES

$$\cos(\theta+\phi) + i\sin(\theta+\phi) = [\cos\theta + i\sin\theta][\cos\phi + i\sin\phi]$$

$$= \cos\theta\cos\phi - \sin\theta\sin\phi +$$

$$+ i[\sin\theta\cos\phi + \sin\phi\cos\theta]$$

ASSOCIATING THE REAL & IMAGINARY PARTS GIVES

$$\boxed{\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi} \quad \text{QED!}$$

$$\boxed{\sin(\theta+\phi) = \sin\theta\cos\phi + \sin\phi\cos\theta} \quad \text{QED!}$$