8.30) WRITE DOWN THE Θ EQUATION FOR $\ell = m = 1$. VERIFY THAT $\Theta = \sin\theta$ IS A SOLUTION. WRITE THE COMPLETE WAVE FUNCTION FOR $\ell = 1$, $m = \pm 1$. WITH r FIXED, IN WHAT DIRECTION IS $|Y|^2$ A MAXIMUM FOR THESE STATES?

THE Θ EQUATION IS (8.65)

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left[\sin\theta\frac{d\Theta}{d\theta}\right] + \left[\ell(\ell+1) - \frac{m^2}{\sin^2\theta}\right]\Theta = 0 \qquad (1)$$

FOR $\ell = m = 1$, THIS BECOMES

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left[\sin\theta\frac{d\Theta}{d\theta}\right] + \left[2 - \frac{1}{\sin^2\theta}\right]\Theta = 0 \qquad (2)$$

SEE IF $\Theta = \sin\theta$ IS A SOLUTION:

DEALING WITH THE FIRST TERM,

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left[\sin\theta\frac{d\Theta}{d\theta}\right] = \frac{1}{\sin\theta}\frac{d}{d\theta}\left[\sin\theta\frac{d}{d\theta}\sin\theta\right]$$

$$= \frac{1}{\sin\theta}\frac{d}{d\theta}\left[\sin\theta\cos\theta\right]$$

$$= \frac{1}{\sin\theta}\left(\cos^2\theta - \sin^2\theta\right)$$

BACK TO THE FULL EQUATION (2)

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left[\sin\theta\frac{d\Theta}{d\theta}\right] + \left[2 - \frac{1}{\sin^2\theta}\right]\Theta =$$

$$\frac{1}{\sin\theta}\left(\cos^2\theta - \sin^2\theta\right) + \left(2\sin\theta - \frac{1}{\sin\theta}\right) \stackrel{?}{=} 0$$

MULTIPLYING THROUGHT BY $\sin\theta$ GIVES

$$\cos^2\theta - \sin^2\theta + 2\sin^2\theta - 1 \stackrel{?}{=} 0$$

$$\cos^2\theta + \sin^2\theta - 1 \stackrel{?}{=} 0$$

$$\boxed{1 - 1 = 0} \quad \text{QED!}$$

YES! $\Theta = \sin\theta$ IS A SOLUTION.

→

8.30) CONTINUED

THE COMPLETE WAVE FUNCTION FOR $\ell = m = \pm 1$ IS

$$\psi(r,\theta,\phi) = R(r)(\sin\theta)e^{\pm i\phi} \qquad \text{(AFTER 8.64)}$$

THIS GIVES A PROBABILITY DENSITY OF $(|e^{\pm i\phi}|^2 = 1)$

$$|\psi|^2 = |\psi^*\psi| = |R^2(r)|\sin^2\theta$$

FOR $R(r)$ HELD CONSTANT, THE MAXIMUM WILL BE WHERE $\sin^2\theta$ IS A MAXIMUM

$$\Rightarrow |\psi|^2_{MAX} \Rightarrow \sin^2\theta = \text{MAXIMUM} = 1$$

$$\Rightarrow \theta = \frac{\pi}{2} \Rightarrow \text{MAXIMUM ON THE } x\text{-}y \text{ PLANE}$$