8.40) THE RADIAL FUNCTIONS FOR $\ell = n-1$ ARE REASONABLY SIMPLE,

$$R(r) = A r^{n-1} e^{-r/na_B} \qquad (8.108)$$

a) WRITE OUT THE RADIAL SCHRÖDINGER EQUATION FOR THIS.

b) VERIFY THAT 8.108 IS A SOLUTION IFF $E = -E_R/n^2$

a) THE RADIAL SCHRÖDINGER EQUATION, 8.72 IS

$$\frac{\partial^2}{\partial r^2}(rR) = \frac{2m_e}{\hbar^2}\left[-\frac{ke^2}{r} + \frac{\ell(\ell+1)\hbar^2}{2m_e r^2} - E\right](rR) \qquad (8.72)$$

FOR $\ell = n-1$

$$\ell(\ell+1) = (n-1)(n-1+1) = n(n-1)$$

$$\boxed{\Rightarrow \frac{\partial^2}{\partial r^2}(rR) = \frac{2m_e}{\hbar^2}\left[-\frac{ke^2}{r} + \frac{n(n-1)\hbar^2}{2m_e r^2} - E\right](rR)} \qquad (1)$$

b) SHOW $R(r) = A r^{n-1} e^{-r/na_B}$ SOLVES THIS

THE LEFT SIDE BECOMES

$$\frac{\partial^2}{\partial r^2}(rR) = \frac{\partial^2}{\partial r^2}\left(A r^{(1+n-1)} e^{-r/na_B}\right)$$

$$= A\frac{\partial}{\partial r}\left[n r^{(n-1)} - \frac{r^n}{na_B}\right]e^{-r/na_B}$$

$$= A\left[\left(n(n-1)r^{(n-2)} - \frac{\cancel{n} r^{(n-1)}}{\cancel{n} a_B}\right) - \frac{1}{na_B}\left(nr^{(n-1)} - \frac{r^n}{na_B}\right)\right]e^{-r/na_B}$$

$$= A\left[n(n-1)r^{(n-2)} - \left(\frac{1}{a_B} + \frac{1}{a_B}\right)r^{(n-1)} + \frac{r^n}{n^2 a_B^2}\right]e^{-r/na_B}$$

$$= A\left[n(n-1)r^{(n-2)} - \frac{2}{a_B}r^{(n-1)} + \frac{r^n}{n^2a_B^2}\right]e^{-r/na_B} \qquad (2)$$

THE RIGHT-HAND SIDE, REPLACING $a_B = \frac{\hbar^2}{m_e ke^2}$

$$\left[-\frac{2}{a_B r} + \frac{n(n-1)}{r^2} - \frac{2m_e}{\hbar^2}E\right]A r^{(1+n-1)} e^{-r/na_B} =$$

$$\left[-\frac{2}{a_B}r^{(n-1)} + n(n-1)r^{(n-2)} - \frac{2m_e r^n}{\hbar^2}E\right]A e^{-r/na_B} \longrightarrow$$

8.40) CONTINUED

SETTING THE RIGHT AND LEFT SIDES EQUAL GIVES

$$\left[\cancel{n(n-1)r^{(n-2)}} - \cancel{\frac{2}{a_B}r^{(n-1)}} + \frac{r^n}{n^2 a_B^2}\right]\cancel{Ae^{-r/na_B}} =$$

$$\left[\cancel{n(n-1)r^{(n-1)}} - \cancel{\frac{2}{a_B}r^{(n-1)}} - \frac{2m_e r^n}{\hbar^2}E\right]\cancel{Ae^{-r/na_B}}$$

THUS

$$\frac{\cancel{r^n}}{n^2 a_B^2} = -\frac{2m_e \cancel{r^n}}{\hbar^2}E$$

$$\Rightarrow E = \frac{\hbar^2}{2m_e n^2 a_B^2}$$

RECALLING THAT

$$E_R = \frac{ke^2}{2a_B} \quad \text{AND} \quad a_B = \frac{\hbar^2}{m_e ke^2} \Rightarrow \frac{\hbar^2}{m_e} = ke^2 a_B$$

$$\Rightarrow E = \frac{ke^2 \cancel{a_B}}{2n^2 a_B^{\cancel{2}}} = \frac{ke^2}{2n^2 a_B}$$

$$\Rightarrow E = \frac{ke^2}{2a_B}\frac{1}{n^2}$$

$$\Rightarrow \boxed{E = \frac{E_R}{n^2}} \quad \text{QED!}$$