8.44) a) VERIFY THAT THE FOLLOWING SOLVES THE RADIAL EQUATION FOR $n=2$ (8.107)

$$R_{2S}(r) = A\left(2 - \frac{r}{a_B}\right) e^{\frac{-r}{2a_B}}$$

b) FIND A FROM NORMALIZATION.

THE RADIAL EQUATION IS:

$$\frac{d^2}{dr^2}(rR) = \left[\frac{1}{a_B^2 n^2} - \frac{2}{a_B r}\right](rR) \qquad (8.107)$$

TO SHOW IT'S A SOLUTION, TAKE DERIVATIVES,

$$\frac{d}{dr}(rR) = A\frac{d}{dr}\left[\left(2r - \frac{r^2}{a_B}\right)e^{-\frac{r}{2a_B}}\right]$$

$$= A\left[\left(2 - \frac{2r}{a_B}\right) - \frac{1}{2a_B}\left(2r - \frac{r^2}{a_B}\right)\right]e^{-\frac{r}{2a_B}}$$

$$= A\left(2 - \frac{3r}{a_B} + \frac{r^2}{2a_B^2}\right)e^{\frac{-r}{2a_B}}$$

$$\frac{d^2}{dr^2}(rR) = A\frac{d}{dr}\left(2 - \frac{3r}{a_B} + \frac{r^2}{2a_B^2}\right)e^{-\frac{r}{2a_B}}$$

$$= A\left[\left(0 - \frac{3}{a_B} + \frac{2r}{2a_B^2}\right) - \frac{1}{2a_B}\left(2 - \frac{3r}{a_B} + \frac{r^2}{2a_B^2}\right)\right]e^{-\frac{r}{2a_B}}$$

$$= A\left[-\frac{4}{a_B} + \frac{5r}{2a_B^2} - \frac{r^2}{4a_B^3}\right]e^{-\frac{r}{2a_B}}$$

SUBSTITUTE INTO (8.107) $\longrightarrow$ FOR $n=2$

$$\cancel{A}\left[\frac{5r}{2a_B^2} - \frac{4}{a_B} - \frac{r^2}{4a_B^3}\right]\cancel{e^{-\frac{r}{2a_B}}} = \left[\frac{1}{4a_B^2} - \frac{2}{a_B r}\right]\cancel{A}r\left(2 - \frac{r}{a_B}\right)\cancel{e^{-\frac{r}{2a_B}}}$$

$$\frac{5r}{2a_B^2} - \frac{4}{a_B} - \frac{r^2}{4a_B^3} = \frac{r}{2a_B^2} - \frac{4\cancel{r}}{a_B\cancel{r}} - \frac{r^2}{4a_B^3} + \frac{2r^{\cancel{2}}}{a_B^2\cancel{r}}$$

$$\cancel{\frac{5r}{2a_B^2}} - \cancel{\frac{4}{a_B}} - \cancel{\frac{r^2}{4a_B^3}} = \cancel{\frac{5r}{2a_B^2}} - \cancel{\frac{4}{a_B}} - \cancel{\frac{r^2}{4a_B^3}} \longrightarrow$$

MIQUELRIUS

8.44) CONTINUED

THUS
$$0 = 0$$

AND $R_{2s} = A\left(2 - \frac{r}{a_B}\right) e^{-\frac{r}{2a_B}}$ $\underline{\underline{\text{IS}}}$ A SOLUTION!

b) FIND A FROM NORMALIZATION

$$\Rightarrow \int_0^\infty P(r)\,dr = \int_0^\infty 4\pi r^2 |R(r)|^2\,dr = 1$$

THUS
$$\int_0^\infty 4\pi r^2 \left[A^2\left(2 - \frac{r}{a_B}\right)^2 e^{-\frac{r}{a_B}}\right] dr =$$

$$\Rightarrow \quad = 4\pi A^2 \int_0^\infty r^2\left(4 - \frac{4r}{a_B} + \frac{r^2}{a_B^2}\right) e^{-\frac{r}{a_B}}\,dr$$

$$= 4\pi A^2 \int_0^\infty \left[4r^2 e^{-\frac{r}{a_B}} - \frac{4r^3}{a_B} e^{-\frac{r}{a_B}} + \frac{r^4}{a_B^2} e^{-\frac{r}{a_B}}\right] dr$$

USE TZD II p 683 AND CRC # 369 p. 232

$$\int_0^\infty x^n e^{-\frac{x}{b}}\,dx = n!\,b^{(n+1)}, \quad b = a_B \text{ HERE}$$

THUS
$$\int_0^\infty P(r)\,dr = 4\pi A^2\left[\overset{n=2}{4(2)(a_B)^3} - \overset{n=3}{\frac{4}{a_B}(6)(a_B)^4} + \overset{n=4}{\frac{1}{a_B^2}(24)(a_B)^5}\right]$$

$$= 4\pi A^2\left[8a_B^3 \cancel{- 24a_B^3 + 24a_B^3}\right]$$

$$= 32\pi A^2 a_B^3 = 1 \longleftarrow \text{TO NORMALIZE}$$

$$\Rightarrow \boxed{A = \frac{1}{\sqrt{32\pi a_B^3}}} \Rightarrow \boxed{R_{2s}(r) = \frac{1}{\sqrt{32\pi a_B^3}}\left(2 - \frac{r}{a_B}\right) e^{-\frac{r}{a_B}}}$$

FULL R_{2s} EXPRESSION.