

11.5) USING $P = bk^l q^m a^n c^p$ AND DIMENSIONAL ANALYSIS, DERIVE

$$P = \frac{2kq^2 a^2}{3c^3} \quad (11.1)$$

LOOK AT UNITS:

$$P = bk^l q^m a^n c^p$$

← SPEED OF LIGHT $\sim \frac{m}{s}$
 ← ACCELERATION $\sim \frac{m}{s^2}$
 ← CHARGE $\sim C$ (COULOMBS)
 ← COULOMB CONSTANT $\sim \frac{N \cdot m^2}{C^2}$
 ← CONSTANT, NO UNITS

Thus

$$P = b \left(\frac{N \cdot m^2}{C^2} \right)^l (C)^m \left(\frac{m}{s^2} \right)^n \left(\frac{m}{s} \right)^p$$

$$P \sim N^l m^{2l+n+p} C^{-2l+m} s^{-2n-p}$$

POWER IS IN WATTS

$$\Rightarrow P \sim \frac{J}{s} \sim \frac{N \cdot m}{s} = N^1 m^1 s^{-1}$$

EQUATING THE POWER ON THE UNITS

$$N: l = 1$$

$$m: 2l + n + p = 1 \Rightarrow 2 + n + p = 1 \Rightarrow n + p = -1$$

$$s: -2n - p = -1 \Rightarrow n + (n + p) = 1 \Rightarrow n = 2$$

$$\text{"} \Rightarrow 4 + p = 1 \Rightarrow p = -3$$

$$C: -2l + m = 0 \Rightarrow -2 + m = 0 \Rightarrow m = 2$$

Thus

$$P = bk^l q^m a^n c^p = bk^2 q^2 a^2 c^{-3} = \left| \frac{bkq^2 a^2}{c^3} = P \right|$$

where $b = \frac{2}{3}$ so it's within a constant