9.1 a) USE EQUATION (9.5)

$$g(v_x)\,dv_x = \sqrt{\frac{\beta m}{2\pi}}\, e^{-\frac{1}{2}\beta m v_x^2}\, dv_x \qquad (9.5)$$

TO SHOW THE ONE-DIMENSIONAL rms SPEED IS

$$v_{x,rms} = \left(\overline{v_x^2}\right)^{1/2} = \left(\frac{kT}{m}\right)^{1/2}.$$

b) SHOW THAT EQUATION (9.5) CAN BE RE-WRITTEN AS

$$g(v_x)\,dv_x = \frac{1}{\sqrt{2\pi}\, v_{x,rms}}\, e^{-\frac{1}{2} v_x^2 / v_{x,rms}^2}\, dv_x$$

THE rms SPEED IS THE AVERAGE WEIGHTED BY v^2

$$\overline{v_x^2} = \int_{-\infty}^{\infty} v^2 g(v_x)\, dv_x = \sqrt{\frac{\beta m}{2\pi}} \int_{-\infty}^{\infty} v_x^2 e^{-\frac{1}{2}\beta m v_x^2}\, dv_x$$

SINCE THIS IS AN EVEN FUNCTION OVER SYMMETRIC LIMITS

$$\overline{v_x^2} = 2\sqrt{\frac{\beta m}{2\pi}} \int_0^{\infty} v_x^2 e^{-\frac{1}{2}\beta m v_x^2}\, dx$$

$\lambda = \frac{1}{2}\beta m$

p 683 IN TZDII GIVES

$$\int_0^{\infty} x^2 e^{-\lambda x^2}\, dx = \sqrt{\frac{\pi}{16\lambda^3}} = \sqrt{\frac{\pi}{16}\left(\frac{8}{\beta^3 m^3}\right)} = \sqrt{\frac{\pi}{2\beta^3 m^3}}$$

$$\Rightarrow \overline{v_x^2} = 2\sqrt{\frac{\beta m}{2\pi}}\sqrt{\frac{\pi}{2\beta^3 m^3}} = \sqrt{\frac{1}{\beta^2 m^2}} = \frac{1}{\beta m} = \frac{kT}{m}$$

THEREFORE

$$\boxed{v_{x,rms} = \sqrt{\overline{v^2}} = \sqrt{\frac{kT}{m}}}$$

QED!

b) REPLACING $\overline{v_x^2} = \frac{1}{\beta m}$ AND $v_{x,rms} = \sqrt{\frac{1}{\beta m}}$

$$\boxed{g(v_x)\,dv_x = \frac{1}{\sqrt{2\pi}\, v_{x,rms}}\, e^{-\frac{1}{2}(v_x/v_{x,rms})^2}\, dx}$$

QED!