

9.24 What fraction of e⁻ in a good conductor have energies between $0.95E_F$ and E_F at $T=0$?

The number of electrons with $E < \text{energy} < E + dE$ is

$$n(E) dE = \frac{3N}{2} \frac{E_F^{-3/2}}{E} E^{1/2} \frac{1}{e^{(E-E_F)/kT} + 1}$$

For $E < E_F$ and $T=0$, the exponential factor goes to $e^{-\infty} \approx 0$ so the Fermi-Dirac factor goes to one. Thus

$$N(0.95E_F - E_F) = \frac{3N}{2} \int_{0.95E_F}^{E_F} \frac{E_F^{-3/2}}{E} E^{1/2} dE$$

$$\begin{aligned} N_{0.95} &= \frac{3N}{2} E_F^{-3/2} \left(\frac{2}{3} E^{3/2} \right) \Big|_{0.95E_F}^{E_F} \\ &= N \frac{1}{E_F^{3/2}} E_F^{3/2} (1 - 0.95^{3/2}) \end{aligned}$$

$$N_{0.95} = 0.074 N$$

The ratio of $N_{0.95}$ to N is

$$\frac{N_{0.95}}{N} = 0.074 \Rightarrow \frac{N_{0.95}}{N} = 7.4\%$$