

HOMEWORK SET 9: EQUILIBRIUM & STABILITY
Due Wednesday, September 29, 2021

PROBLEMS FROM TM5.

1) 2-43 A particle is under the influence of a force $F = -kx + kx^3/\alpha^2$. Where k and α are constants and k is positive. Determine $U(x)$ and discuss the motion using computer plots of $F(x)$ and $U(x)$. What is the situation and what will occur when $E = (1/4)k\alpha^2$?

2) 2-47 Consider a particle moving in the region $x > 0$ under the influence of the potential

$$U(x) = U_0 \left(\frac{\alpha}{x} + \frac{x}{\alpha} \right)$$

where $U_0 = 1$ J and $\alpha = 2$ m. Plot the potential (using a computer), find the equilibrium points, and determine whether they are maxima or minima.



CALCULUS AND ALGEBRA PRACTICE

3) Show all the steps to get from setting

$$\left. \frac{dU(x_1)}{dx_1} \right|_{x_1=x_0} = \frac{d}{dx_1} \left\{ -m_1 g x_1 - m_2 g \sqrt{\left(\frac{b-x_1}{2} \right)^2 - d^2} \right\} = 0$$

to

$$x_0 = b - \frac{4m_1 d}{\sqrt{4m_1^2 - m_2^2}}$$

(THE POINT IS FOR YOU TO EXPLAIN TO SOMEONE WHO DOESN'T KNOW HOW TO DO IT HOW IT'S DONE, NOT SHOW ME YOU CAN DO IT)

4) Show all the steps to get from

$$\left. \frac{d^2U(x_1)}{dx_1^2} \right|_{x_1=x_0} = \frac{d}{dx_1} \left\{ -m_1 g + \frac{m_2 g (b-x_1)}{4 \sqrt{\left(\frac{b-x_1}{2} \right)^2 - d^2}} \right\}$$

to

$$\left. \frac{d^2U(x_1)}{dx_1^2} \right|_{x_1=x_0} = \frac{g(4m_1^2 - m_2^2)^{3/2}}{4m_2^2 d}$$

by substituting the value of x_1 from above and simplifying.