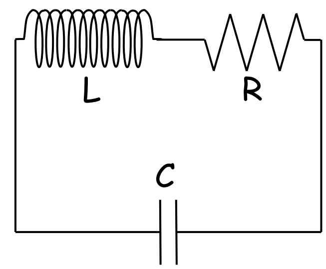
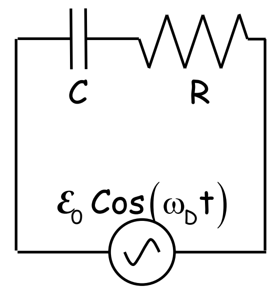
Homework Set 14: The Mechanical-Electrical Connection
Due Monday, October 23, 2023

Problems From MT3 (Not in TM5 ... text from MT3 on T :\O'Donoghue\Mechanics\ClassNotes) .



1) *3-22* Show that for an RLC circuit in which the resistance is small, the logarithmic decrement of the oscillations is approximately $\pi R\sqrt{C/L}$.
The logarithmic decerement is defined on p. 111 of TM5 shown below.

2) *3-23* Compute the oscillation frequencies, periods and amplitude after 2 periods (as a fraction of A_0) for the circuit shown for L = 0.01 H, C = 10 μF, and R = 10 Ω.



3) *3-24* An electrical circuit consists of a resistor R and a capacitor C connected in series to a source of alternating emf. Find expressions for the charge and current as a function of time and show that the current decreases to zero as the frequency of the alternating emf approaches zero.
Solve a differential equation similar to that in Example 3.5 (above Figure 3-18 on p. 125). Guess a solution that is a sum of a cosine and a sine (instead of using a phase constant), then force it to work ... look back at how we solved the driven harmonic oscillator ... but use $Q(t) = A\cos(\omega_D t) + B\sin(\omega_D t)$. I get,

$$Q(t) = \frac{C\mathcal{E}_0}{\omega_D^2 R^2 C^2 + 1}\left[\cos(\omega_D t) + \omega_D RC \sin(\omega_D t)\right]$$

$$I(t) = \frac{\omega_D C\mathcal{E}_0}{\omega_D^2 R^2 C^2 + 1}\left[\omega_D RC \cos(\omega_D t) - \sin(\omega_D t)\right]$$

3.5 DAMPED OSCILLATIONS 111

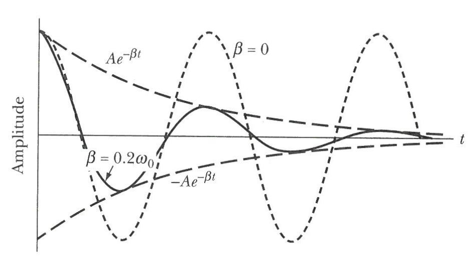


FIGURE 3-7 The underdamped motion (solid line) is an oscillatory motion (short dashes) that decreases within the exponential envelope (long dashes).

The ratio of the amplitudes of the oscillation at two successive maxima is

$$\frac{Ae^{-\beta T}}{Ae^{-\beta(T+\tau_1)}} = e^{\beta\tau_1} \tag{3.42}$$

where the first of any pair of maxima occurs at $t = T$ and where $\tau_1 = 2\pi/\omega_1$. The quantity $\exp(\beta\tau_1)$ is called the **decrement** of the motion; the logarithm of $\exp(\beta\tau_1)$—that is, $\beta\tau_1$—is known as the **logarithmic decrement** of the motion.

Unlike the simple harmonic oscillator discussed previously, the energy of the damped oscillator is not constant in time; rather, energy is continually given up to the damping medium and dissipated as heat (or, perhaps, as radiation in the form of fluid waves). The rate of energy loss is proportional to the square of the velocity (see Problem 3-11), so the decrease of energy does not take place uniformly. The loss rate will be a maximum when the particle attains its maximum velocity near (but not exactly at) the equilibrium position, and it will instantaneously vanish when the particle is at maximum amplitude and has zero velocity. Figure 3-8 shows the total energy and the rate of energy loss for the damped oscillator.

"Got any 25-amp fuses?"