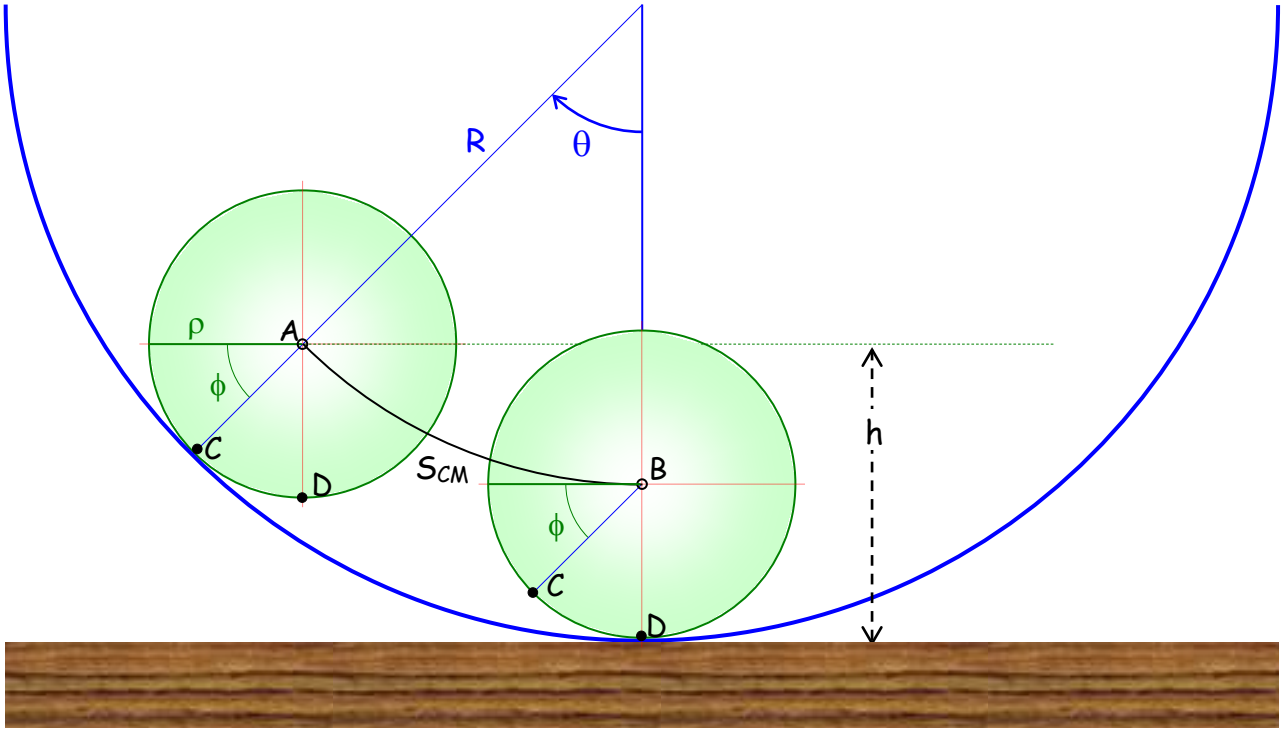
Sphere Rolling in a Circular Bowl (TM5 Pr. 7-3)

Why does the constraint $S_{CM} = S_{Roll}$ mean that $(R - \rho)\theta = \rho\phi$?

Say the sphere rolls through one revolution in moving the CM from point A to point B.



The CM moves along the arc, $S_{CM} = (R - \rho)\theta$ and point C moves around the circumference of the sphere, $S_C = 2\pi\rho$. This is a shorter distance than the arc length between the points of contact, $S_{Bowl} = R\theta$. This is because the contact point with the bowl changes even though the sphere has had a full rotation.

If it were rolling on a flat surface, a full rotation would put point C as the contact point and it would have to roll a distance $S_D = \rho\theta$ to make D the contact point. The total distance rolled would then be, $S_{Bowl} = S_{C\ to\ D} = 2\pi\rho. + \rho\theta$ since one full rotation roll would put C back as the contact point so to make D the final contact point, it has to roll a little bit farther