## Motion with Constant Acceleration

Fall 2023

## Introduction

The purpose of this experiment is to determine the acceleration, $a_{o}$ of a cart on a sloped track and see if the acceleration is constant. You will see the extent to which the data are consistent with the kinematic expression $x=x_{o}+v_{o} t+1 / 2 a_{o} t^{2}$, and how close the measured acceleration is to the expected acceleration $a_{x}=g \sin \theta$, where $\theta$ is the angle of the track. You will also see if the mass of the cart affects the acceleration.


## Experiment

1. Write the letter and number of the track and cart that you are using on the apparatus sketch in your journal.
2. Prediction: You would generally presume that the measured acceleration, $a_{0}$ of the cart as it rolls down the track is the same as the expected acceleration on a frictionless track, $a_{\mathrm{x}}$ if you were doing this as a homework problem. What do you think you will find for the relationship between $a_{0}$ and $a_{\mathrm{x}}$ for your cart after you have performed this experiment in lab today: will they be equal, or will one quantity be greater than or less than the other? State your prediction in your journal, and briefly explain your reasoning.
3. Prepare a data table in your journal with the columns shown below. You will be collecting a lot of data, so start the table at the top of a separate page to give yourself plenty of room.

| Initial Position, <br> $x_{i}(\mathrm{~cm})$ | Final Position, <br> $x_{f}(\mathrm{~cm})$ | Time, <br> $t(\mathrm{sec})$ | Average Time, <br> $\langle t\rangle(\mathrm{sec})$ | Distance Fallen, <br> $x=\left\|x_{f}-x_{i}\right\|(\mathrm{cm})$ |
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4. Set the photogate timer to "pulse" mode with the memory switch to the "On" position and record these settings in your journal. The photogate is positioned so that the timer is tripped when the front edge of the cart is at the 110.0 cm mark of the track. Check this by holding the front edge of the cart at 110.0 cm ; the red LED on the top of the gate should turn on. Alert your instructor if this position is off by more than 2 mm .
5. You will measure the time, $t$ for the cart to fall a known distance, $x\left(=x_{f}-x_{i}\right)$. Hold the cart with your pen or pencil so that the front edge of the cart is at the desired $x_{i}$. Press and hold the white Start button on the photogate timer (don't release it yet!). Release the white Start button on the timer at the same moment that you release the cart; the timer will stop when the cart passes through the photogate, measuring the time for each value of $x$ you choose. Practice the procedure a few times until you feel comfortable with it.
6. Begin at $x_{i}=15.0 \mathrm{~cm}$, which will give you $x=95.0 \mathrm{~cm}$, the largest distance possible on the tracks you are using. Collect a set of values of $t$ for this $x$ (four or five measurements should give you reliable data), then calculate the average of your times, $\langle t\rangle$. Exclude from your calculation those measurements of $t$ which are obviously incorrect: draw a light line through these excluded numbers. Be sure to record all digits from the photogate timer; don't round these measured values!
7. Create a table in Excel with the average time, $\langle t\rangle$ in the first column, and the distance fallen, $x$ in the second. Recall that Excel needs at least three points to correctly create an $x-y$ scatter plot! You already have two data points (your largest $x$, and $x=0$ ), so you need to measure times for one more value of $x$ (choose a distance 10 cm shorter than the max). Create a graph in Excel, plotting $x$ on the vertical axis and $\langle t\rangle$ on the horizontal axis.
8. Collect time measurements for intermediate distances at 10 cm intervals, being sure to record all your data in your journal and plot each point as it is calculated. Also measure times for $x=5.0 \mathrm{~cm}$, the shortest practical distance. Record the data in your journal and in Excel in the same order it is collected; it will still plot correctly.

Since you are collecting a lot of data today, your data table will become crowded and difficult to read if you underline each entry. Instead, draw a horizontal line in your table only to separate times measured at different distances.
9. Finally, measure ' $L$ ' and ' $h$ ' as indicated in the sketch and calculate the track angle $\theta$ (set your calculator to degrees, not radians!) Calculate the expected acceleration along the $x$-axis using $a_{x}=g \sin \theta$.

## Analysis

The cart was released at $x=0$ with no initial velocity, so $x_{0}$ and $v_{0}$ are zero, and constant acceleration would make $x=1 / 2 a_{o} t^{2}$ (a parabola). You have a measured set of $\{\langle t\rangle, x\}$ pairs. Comparing your data points with a parabola will allow you to see if these data are consistent with the theory.
10. Add a best-fit curve (a $2^{\text {nd }}$ order polynomial - be sure to display the equation!) on your Excel graph. Print your graph and record the values of the coefficients $A\left(=1 / 2 a_{0}\right), B\left(=v_{0}\right)$, and $C\left(=x_{0}\right)$ in your journal. Be sure to solve for the measured acceleration, $a_{0}$.
11. As soon as you have calculated $a_{0}$, go to the data table containing the class results on the blackboard and record your measured acceleration, $a_{0}$ next to the mass of your cart ("heavy" or "light"). All the tracks in the lab are inclined at approximately the same angle, which allows you to see any mass-related effects on acceleration.

## Discussion

- Begin by creating a summary table of your measured and expected results of acceleration, $a_{o}$ and $a_{x}$; initial position, $x_{o}$; initial velocity, $v_{o}$; and the measured track angle, $\theta$.
- Calculate the $\%$ difference between $a_{o}$ and $a_{x}$. How well do they agree with each other? Check with your instructor if the difference is more than $10 \%$.
- How well do your measured and expected values for $x_{o}$ and $v_{o}$ agree? Don't bother calculating \% difference here; when one value is expected to be zero, you will always get a 200\% difference!
- Earlier, you made a prediction about the relationship between $a_{o}$ and $a_{x}$. Does your prediction match your results? What are some sources of error that might affect the acceleration?
- Hint: There are two possible sources of error that can affect acceleration - one is obvious, the other will require some additional thought. Use your measured $a_{o}$ to calculate the actual track angle that would
give you the measured acceleration: $\theta_{\text {actalal }}=\sin ^{-1}\left(a^{2} / \mathrm{g}\right)$. How does this angle compare with the one you measured in step 9 ? What might cause uncertainty in your angle measurement? Hint: Look carefully at your lab bench.
- Did your cart accelerate at a constant rate, as predicted by the theory, $x=x_{o}+v_{o} t+1 / 2 a_{o} t^{2}$ (i.e., is your graph a parabola? How do you know?)
- Look at the table on the board and discuss your results in comparison with the other groups. Is there any significant difference in acceleration of heavy carts vs. light carts? Be sure to include the board table in your journal, indicate the track that you used, and whether your cart was heavy or light.

