

## Standing Waves: The Simple Guitar

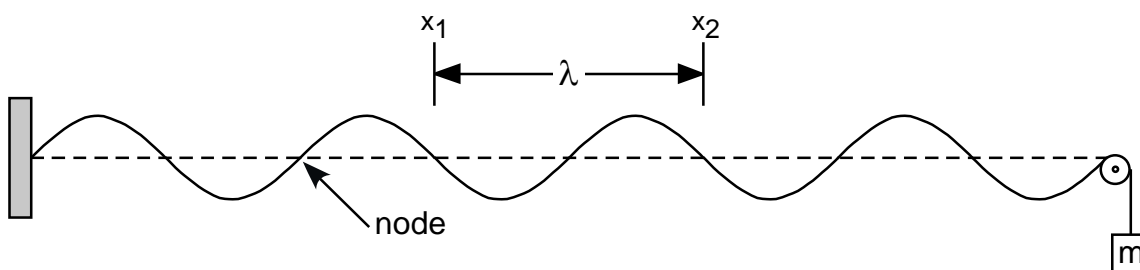
Fall 2011

### Introduction

In this experiment you will examine the relationship between tension, wavelength and velocity of waves on a string. You will also calculate the *linear density*,  $\mu$ , of the string.

### Theory

Waves on a string clamped at both ends appear to stand still if half their *wavelength* just fits the length of the string between the clamps.



The following quantities can be measured directly: The *wavelength*,  $\lambda$  (the distance between **three** consecutive 'nodes'); the *frequency*,  $f$  of the wave (for this experiment,  $f = 60$  Hz); and the *string tension*,  $\mathbf{T}$ .

From the frequency and wavelength you can calculate the *wave velocity*,  $v = f \cdot \lambda$ . Applying Newton's law to a string gives a theoretical connection between wave velocity, the tension on the string, and the *linear density*,  $\mu$ .

$$v = \sqrt{\frac{T}{\mu}}$$

This expression can be written as  $v = \frac{1}{\sqrt{\mu}}\sqrt{T} = a\sqrt{T}$ , where  $a = \frac{1}{\sqrt{\mu}}$ . The units for  $\mu$  are *mass per length*. You will analyze your data using KaleidaGraph to calculate the value of  $\mu$  for your string.

### Experiment

1. Create a data table with the following headers:

m (kg)	$x_1$ (m)	$x_2$ (m)	$\lambda =  x_2 - x_1 $ (m)	$v = f \cdot \lambda$ (m/s)	$T = mg$ (N)
-----------	--------------	--------------	--------------------------------	--------------------------------	-----------------

2. Suspend 500 g from the string (*this includes the 50 g hanger!*), and turn on the string vibrator. A wave should be visible along the length of the string; if the wave is not visible, *gently* drag the edge of a ruler along the right side of the string until 3 consecutive nodes appear. Don't press too hard on the string, or you'll change the tension!
3. Measure the wavelength  $\lambda$  (*the distance between three consecutive nodes*). Use this measurement to determine the wave velocity. Choose nodes near the center of the string; don't use the nodes that appear at both ends of the string. Calculate the string tension  $\mathbf{T}$  in the string, using  $g = 9.80 \text{ m/s}^2$ .

4. We'll be using a different program to plot our data this week; instructions on using Kaleidagraph are found in the "Analysis" section below. After you've calculated the tension, you can begin a Kaleidagraph plot of wave velocity as a function of string tension. Unlike Excel, Kaleidagraph will add points to the graph as they are entered. *Remember to plot your points as you collect your data!*
5. Repeat the measurements of  $\lambda$ ,  $v$ , and  $T$  for other tensions down to as small a value as possible. *You should be able to get at least 7 data points with the masses provided!*
6. A simple method of calculating the linear density would be to determine the mass of your string, and measure its length; dividing the two would give you  $\mu$ . It is difficult to disassemble the apparatus, so you will determine the mass of an identical 'sample' string and measure its length to get a static calculation of  $\mu$ .

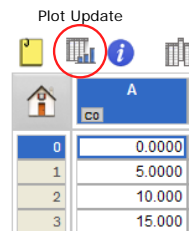
### Analysis



The relationship between wave velocity and string tension follows a power law:  $y = ax^n$ , where the power  $n$  is 0.5 (because of the square root). If we use Excel for our analysis, it will attempt to adjust  $n$  as well as  $a$  to get the best possible fit; unfortunately, anything other than  $n = 0.5$  produces lousy results. So, to improve our analysis, we will use *KaleidaGraph* (icon at right) which is better suited to the task of analyzing our data.

7. Begin creating your table of data in KaleidaGraph by first labeling the data columns (by default, they're labeled A, B, C, etc.). Double-click the column title and change columns A and B to  $T$  and  $v$  (be sure to include units!!). Then enter your  $T$  and  $v$  data (*Note: Do not include the coordinate  $\{0,0\}$  in your table!*).
8. Now you will create a graph of  $v$  vs.  $T$  as follows:
  - i. From the **Gallery** menu, choose **Linear**, then **Scatter**. In the "Plot" dialog that appears, click the radio buttons next to  $T$  and  $v$  to graph them on the  $X$  and  $Y$ -axis, respectively (*now you see why it's important to label the columns!*). Click the **New Plot** button.
  - ii. The first thing you need to do is extend the graph axes so that they include the origin. This is especially important if you are not creating a hand-drawn graph.
    - a. Click the **Plot** menu and choose **Axis Options**.
    - b. In the dialog box that appears (for the x-axis), change the **Minimum** value to 0 (if the value is already 0, uncheck the box in the "Auto" column).
    - c. Click on the **Y** tab, and again change the **Minimum** value to 0. Click **OK** when finished.
  - iii. Now you will create the square root function that you need to analyze your pendulum data:
    - a. Select the **Curve Fit** menu, then choose **General**, and then **Edit General...**
    - b. In the dialog box that appears, click the **Add** button, which will add **New Fit** to the list on the left side of the dialog box.
    - c. Select **New Fit** from the list on the left. You'll notice that the name appears in an entry field in the lower left corner of the dialog box. Select this entry field, and change the name to **Square Root**.
    - d. Click the **Edit...** button in the center of the dialog box.
    - e. In the next window that appears, type the following (add a space after the semicolon):
 
$$a*x^{0.5}; a=1$$
    - f. Click **OK** twice to leave the Curve Fit entry menu.
  - iv. Again select the **Curve Fit** menu, then choose **General**, and then **Square Root**. Click the box next to  $v$ , then click **OK**. A curve will be fit to your data, and a table of results will appear. Click and drag the table to move it on your graph so that it doesn't obscure the curve (*Note: if the table of data does not appear, click the **Plot** menu, and select **Display Equation***).

- v. If you add (or remove) data points, be sure to click the “Plot Update” button, located on the toolbar in the Data window, as shown at right.
- vi. A couple of final adjustments before printing your graph:
- Turn off the legend by choosing **Plot, Display Legend**.
  - Create a proper graph title (it probably says “Data 1” now); double-click the existing title and edit it. Press Enter, *type your names* and click **OK**.
  - Choose **Print Graphics** from the **File** menu to print the graph (*Don't use the print button on the toolbar – it might print too many copies!*).
9. Use your measured value of the coefficient  $a$  to calculate an experimental value for the mass per unit length of the string,  $\mu$ .



## Discussion

- Report your two values for  $\mu$ . Do they agree with each other? Discuss what may have contributed to the difference between values.