

Pressure and Buoyant Force

Fall 2023

Introduction

In this experiment, you will use the pressure–depth relation to calculate the pressure at some depth below a water surface, and you will use Archimedes' principle to calculate the buoyant force acting on an object.

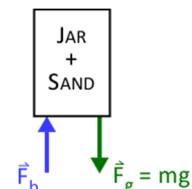
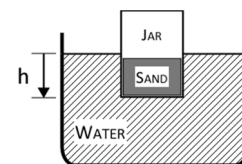
Part I. The Pressure–Depth Relation:

Theory

A body, which is less dense than water, placed on a water surface will sink into the liquid until the body experiences a buoyant force, \vec{F}_b that equals its weight, \vec{F}_g . This means that when the body floats, its weight and the buoyant force are the same in magnitude but opposite in direction. You will use a cylinder (a plastic jar) so that the buoyant force due to the fluid acts only on the bottom of the cylinder if the jar floats vertically.

Once you know the force which acts on the bottom of the jar and the area of the bottom you can find the **pressure** on the bottom of the jar. Note that this pressure is due only to the weight of the jar and that the atmosphere makes no contribution. From the definition of pressure, we have:

$$\text{Pressure} \equiv \frac{\text{force}}{\text{area}} = \frac{F_b}{A} = \frac{F_g}{A} = \frac{mg}{A} \quad \left\{ \text{units} : \frac{N}{m^2} = \text{Pascal} \right\}$$



This gives the pressure of the water on the cylinder bottom at that depth below the surface.

We can also use the pressure–depth relation to calculate the pressure some distance below the fluid surface: $P = \rho_w gh$, where ρ_w (the Greek letter “rho”) is the density of water (998.0 kg/m^3), and h is the depth in the liquid.

Experiment:

1. Create a data table in your journal with the following headers, writing the headers on the long edge of a new page (turn the paper sideways!):

Jar (Large/small)	D_{top} (m)	D_{bottom} (m)	$\langle D \rangle$ (m)	A (m^2)	h (m)	m (kg)	F_g (N)	$P_1 = \frac{F_g}{A}$ (Pa)	$P_2 = \rho_w gh$ (Pa)	%Diff P_1 & P_2

2. Use a ruler to measure the outer diameter across the top of the small jar and then across the bottom. The jars are slightly tapered, so you also need to calculate the *average* of these two diameters, $\langle D \rangle$. You can now also calculate the area of the bottom of the jar, A .
3. Place the edge of a piece of tape on the small jar so that it marks about $\frac{1}{2}$ the distance from the bottom. Go to the sink and add sand to the jar so that it floats upright and level at this line (*be careful not to sink your jar!*) Tilt the jar to allow any air trapped beneath it to escape.
4. Remove the jar from the water and dry it off. Measure the distance from your tape mark to the bottom edge of the jar. We need the depth below the water to the bottom *surface* of the jar, so subtract the length of the jar-lip (0.2 cm) and enter this value in your table for the depth, h .
5. Measure the mass, m of the jar containing sand on the electronic balance and calculate its weight, F_g . Calculate the pressure at depth h from the definition of pressure (P_1 in the table), and from the pressure–depth relation (P_2). *Check your calculations and measurements if the percent difference between pressures is more than $\sim 10\%$!*
6. Repeat this procedure using a jar with a larger diameter. *Remove all tape from the jars when finished.*

Part II. Archimedes' Principle:

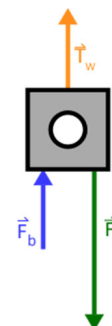
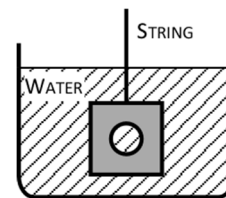
Theory

If a block on a string is suspended in a beaker of water, a free body diagram shows three forces: \vec{F}_g , the weight of the block (which is the same whether suspended in air or under water); \vec{T}_w , the string tension measured with the block in water; and \vec{F}_b , the buoyant force. Applying Newton's 2nd law to this static situation, we see that:

$$F_b + T_w - F_g = 0 \quad \{1\}$$

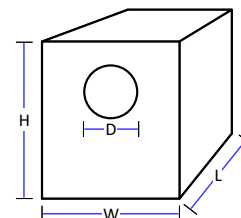
Archimedes' Principle tells us that F_b is also *equal to the weight of the volume of water the object displaces*. The volume of the object (V_{block}) is equal to the volume of the water (V_{water}) displaced. With the density of water assumed to be $\rho_{water} = 998.0 \text{ kg/m}^3$, you can find the mass of this volume of water (m_{water}) and hence the water's weight ($F_{g-water}$). This allows you obtain an independent measurement of buoyant force:

$$\begin{aligned} F_b &= F_{g-water} \\ &= (m_{water})g \\ &= (\rho_{water} \cdot V_{water})g \\ F_b &= (\rho_{water} \cdot V_{block})g \quad \{2\} \end{aligned}$$

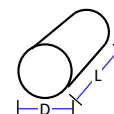


Archimedes' Method of Calculating F_b :

- Use a vernier caliper to measure the dimensions of the aluminum block, recording them in a sketch; recall that the vernier calipers we use measure to 0.002 cm! Use these dimensions to calculate the volume of the block, V_{block} . Your final result will have 4 significant figures – don't round before then!



- The block has a cylindrical hole through it; how will this affect your volume calculation? If you reviewed the vernier caliper instructions, you'll know that there is an easy way to measure the diameter of the hole!



- Use an electronic balance to measure the block's mass and calculate the block's density: $\rho_{block} = \frac{\text{mass}}{\text{volume}}$. Check your volume calculation by comparing your calculated density with the published density of aluminum ($2.700 \times 10^3 \text{ kg/m}^3$). Measure carefully; your calculated density should be **less than 2%** of the published density. If not, recheck your measurements and calculations.

- What does your calculated % difference tell you about the accuracy of your measurements? Include a brief statement in your journal.
- Calculate the *weight of the volume of water* that will be displaced later when the block is submerged, and therefore the buoyant force, using relation {2}. Remember that the volume of water displaced is equal to the volume of the block ($V_w = V_{block}$). *Calculate the buoyant force to 3 significant figures.*

Newton's Method of Calculating F_b :

4. Pick up a force gauge and hold it vertically. Be sure that the gauge reads zero; ask your instructor for assistance if it needs adjustment. Attach the force gauge to the string on the block, and measure its dry weight, W_{block} , in Newtons. *You should notice that the inner scale of the gauge measures force!*
5. Submerge the block *completely* under the water (at least 0.5 cm beneath the surface), making sure the block is suspended level and doesn't hit the bottom of the container. Record the value of the tension, T_{water} (estimate your reading to 0.01 N), and calculate F_b using the derived force expression {1}.
6. Calculate the % diff between your buoyant force values. Check your measurements and calculations if the difference is more than ~5%.
7. The accuracy of the force gauge is sometimes questioned. Develop a simple method of checking the force gauge's accuracy with the equipment available in the lab and then try out your procedure. Record the results in your journal.

Discussion

- Restate your pressure results for the pressure-depth experiment. How well do the pressures agree with each other?
- Briefly describe the two methods used to calculate the pressure on the bottom of the jars.
- Restate the buoyant force results from the Archimedes' Principle experiment. How well did they agree with each other?
- Restate Archimedes' principle, in words. Briefly explain how it was used in this experiment.
- What are some sources of error in each experiment performed today? Also discuss the results you found when checking the accuracy of the force gauge.