Torque & Equilibrium

Fall 20xx

Solutions

Introduction

Today you will calculate the torque necessary to keep an *equal arm balance* in equilibrium. You will also balance and rotate rods with mass, explaining their motion by considering the moment of inertia of the system.

Theory

We have used Newton's Laws to talk about *equilibrium*; equilibrium means that an object is not accelerating because the sum of all the forces acting on the object is zero. In this experiment we introduce the idea of *rotational equilibrium* where an object is not rotating because the sum of the *torques* is zero. Torque can be thought of as a rotational analog of force. The Greek letter $\boldsymbol{\tau}$ (*tau*) is used to represent torque:

$$\tau = Fr\sin\theta \qquad \text{(Eqn. 1)}$$

where F is the applied force; the lever arm, r, is the distance from the pivot (the axis of rotation) to the point where the force is applied; and θ is the angle between r and F (**Figure 1**).

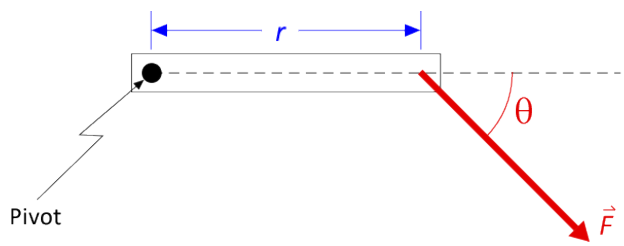

Figure 1: Characteristics of a lever arm

Notice that a longer lever arm results in a larger torque. A larger force also results in a larger torque, as well as an angle θ closer to 90°. Torque can cause an object to rotate in a clockwise or counterclockwise fashion. A clockwise rotation is denoted as a negative torque; a counterclockwise rotation is denoted by a positive torque. *For an object to be in equilibrium, the sum of the torques acting on an object must equal zero.*

Experiment

Part I: Characterizing the Equal Arm Balance

The equal arm balance has seven loops, labeled *3L* to *3R* in **Figure 2**, from which mass can be suspended. Three loops are to the left of center, three to the right of center and one is in the middle. Since we need the length of the lever arm, r, and the angle, θ for our calculations, we will begin by finding those values for our lever arm:

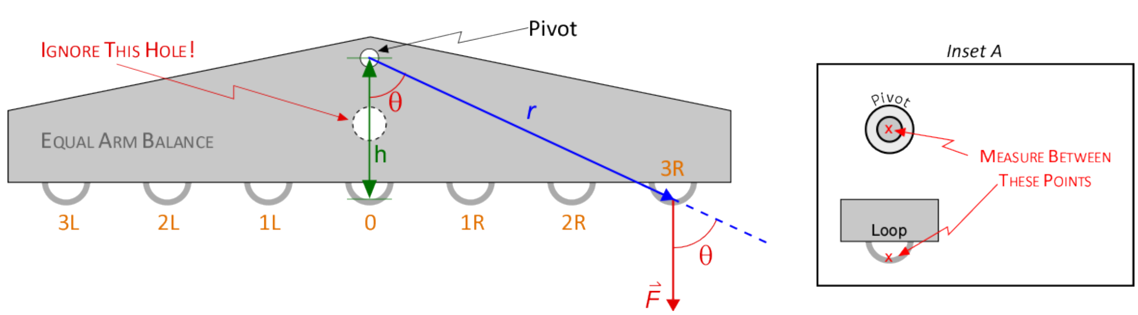

Figure 2: The Equal Arm Balance. *Inset A* shows details of the measurement positions on the pivot and a loop

1. Create **Measurement Table 1** in Excel; <u>note that the measurements of *h* and *r* are in units of meters</u>. You don't need a measurement for hanger position 0:

Excel code: =DEGREES(ACOS(h/r))

Measurement Table 1

Hanger position	h (*m*)	*r* (*m*)	$\theta = \cos^{-1}\left(h/r\right)$
1L	0.0535	0.078	46.7
1R			
2L		0.126	64.9
2R			
3L		0.179	72.6
3R			

2. Remove the equal arm balance from the clamp and use a ruler to measure *h* and the values of *r* for each hanger position 1, 2, and 3 on the left **AND** right side of the pivot (**Figure 2**, *Inset A* – be sure to measure on the <u>front side</u> of the equal arm balance). When your measurements are finished, calculate the angle, θ at which the force is applied to $1/10^{th}$ of one degree. Note that *h* and *r* are measured from the *center of the pivot* to the *part of the loop where the mass hanger makes contact* (Figure 2). Reattach the equal arm balance to the clamp when finished.

   Excel tip: Recall that Excel assumes that you want an angle calculated in radians, but we use degrees in lab. So, the equation to calculate θ will look something like this: `=DEGREES(ACOS(h/r))`, where *h* and *r* represent the cell addresses containing those values.

3. The measurements you just collected are critical, so check them with your instructor before proceeding.

> Note that there is a hole in the equal arm balance between the pivot point and hanger position 0 (SHOWN IN **FIGURE 2**!) Measure *h* from hanger position 0 to the pivot point, **not** the hole!

Part II: Torque and Equilibrium

First you will get a qualitative feel for the torque necessary to balance the arm. Then you will experimentally determine the amount of torque needed to put the lever arm in a state of equilibrium.

4. Add a new sheet to your journal (it should appear <u>**before**</u> the *Data* sheet) and rename the sheet *Predictions*.

5. *Prediction #1*: If you suspend mass on loop 3 on one side of the equal arm balance, from which loop on the *other side* will it be easiest for you to pull straight down to balance the arm: 1, 2 or 3? Explain your choice.

It's easiest to pull from loop 3 on opposite side: long *r* and greater θ means less force is required for same torque.

6. Suspend a mass of 0.250 *kg* (*all the masses in one box!*) on the left side of the equal arm balance at position 3L (remember that the hanger has a mass of 50 *g*!) Now use your hand to balance the arm by pulling *straight down* on the right side, first at position 3R, then position 2R and finally position 1R. You should also try pulling straight down on the loop in the center (position 0).

7. Was your prediction correct? Which loop position required that you pull the hardest to balance the arm? Which required the least effort? Briefly explain the difference, in terms of the length of the lever arm, the angle θ and the force required.

Yes! Since the lever arm length is longer for loop 3 than loop 1, *less force* is needed on loop 3 to provide the <u>*same torque*</u> to balance the suspended mass on the opposite side.

Calculation of Torque

8. *Prediction #2*: If you again suspend mass on loop 3 on the left side of the equal arm balance, which loop on the right side will produce the greatest torque when you balance the apparatus? Or will the torque be the same on all three loops on the right side? Explain your answer.

   Now y ... nd deter ... lo this for two m ...

In equilibrium, the torque will be **the same** on both sides of the equal arm balance so that the net torque is zero.

9. Create *Calculation Table 1* in Excel. This will contain your measurements and torque calculations for the <u>left</u> side of the equal arm balance. Display the value of θ to **<u>3</u> significant figures**:

Excel code: =…`SIN(RADIANS(θ))`

Calculation Table 1 (*Left Side Torque*)

Position – L	r_L (*m*)	θ_L (°)	m_L (*kg*)	τ_L (*N·m*)
3L	0.179	72.6	0.090	0.151
2L	0.126	64.9	0.090	0.100

3L (90 g): $\tau_{3Left} = (m_{Left}\,g)\,r_{Left3}\cdot\sin\theta_3$ = (0.090 *kg*) g (0.179 *m*) sin(72.4°) = 0.151 *N·m*

10. Suspend 90 *g* from loop **3L** on the *left side* of the balance.

11. Use *Calculation Table 1* to calculate the amount of torque that this mass exerts on the left side of the equal arm balance, using *Eqn. 1* and your measurements of the lever arm from Part I: $\tau_{3L} = (mg)r_3\cdot\sin\theta$. *Calculate the torque to **<u>3</u> significant figures**.*

    Excel tip: Since you already calculated θ in degrees, it needs to be converted back to radians so that Excel can calculate the sine of the angle. Therefore, your equation to calculate the torque will contain `SIN(RADIANS(θ))`, where θ is the cell address containing your angle measurement.

12. Create *Calculation Table 2* in Excel. This will contain your measurements and torque calculations for the <u>right</u> side of the equal arm balance:

Calculation Table 2 (*Right Side Torque*)

Position – L	Position – R	r_R (*m*)	θ_R (°)	m_R (*kg*)	τ_R (*N·m*)	%Diff (τ_L & τ_R)
3L	1R	0.078	46.7	0.270	0.150	-0.3%
3L	2R	0.126	64.9	0.135	0.151	0.2%
3L	3R	0.179	72.6	0.090	0.151	0.0%
2L	1R	0.078	46.7	0.180	0.100	-0.5%
2L	2R	0.126	64.9	0.090	0.101	0.0%
2L	3R	0.179	72.6	0.060	0.100	-0.2%

13. Hang enough mass on loop **1R** (*right side*) so that the arm is level.

14. Use *Calculation Table 2* to calculate τ_{1R}, the amount of torque exerted on loop 1R by this mass and then calculate the % diff between this right-side torque and the left-side torque you calculated in Step 11. *You should double-check your measurements if the difference is greater than 5%!*

15. Again, balance the arm by first hanging mass on loop **2R** and then again on loop **3R**. Each time, calculate the right side torques, and calculate the % diff of each with the torque on the left side from Step 11.

16. <u>Repeat the measurements and calculations</u> (steps 10 – 15), this time suspending 90 *g* from loop **2L**.

17. Were you correct with your answer for prediction #2? Briefly discuss your results.

    My prediction was correct – *was yours*? The low %Difference values show that the torque was the same on the left and right sides of the equal arm balance.

Part III: Balancing a Rod with Mass on one End

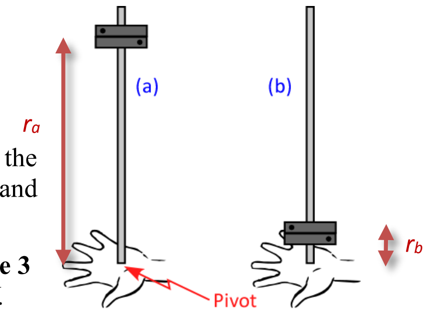

Figure 3: Balancing the rod and masses

You will find a metal rod with two masses at one end on a bench in the lab.

a) *Prediction #3*: Which orientation of the rod will be easier to balance: with the masses far away from your hand (**Figure 3a**) or with the mass near your hand (**Figure 3b**).

b) Try balancing the rod each way – was your prediction correct? Add **Figure 3** to the *Predictions* page of your journal and label *r* for each rod (see part *V. Drawing Arrows and Lines* from Useful Things to Know in Microsoft Excel). Briefly explain the difference between the two orientations using the idea of moment of inertia, *I*:

$$I = \sum_{i=1}^{N} m_i r_i^2$$

The rod is easier to balance in position (a). The moment of inertia is greater in this orientation because the lever arm, ***r*** is longer ($r_a > r_b$ in Figure 3).

c) You will also find a blue and red plastic rod in the lab. *Both rods are of the same mass.* Using one rod at a time, hold it in the center with one hand, and rotate clockwise and counterclockwise (**Figure 4**). Now try it with the other rod, *using the same hand as before*. Which rod is easier to start rotating?

d) Add **Figure 4** to the *Predictions* page of your journal, again labeling *r* for each, and explain why one rod is easier to rotate in terms of the moment of inertia.

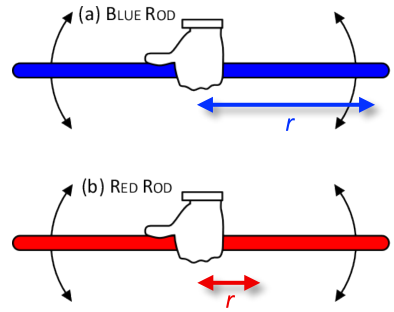

Figure 4: Twirling the (a) blue and (b) red plastic rods

Discussion

The red wand is easier to rotate than the blue. The masses are closer to the center in the red wand, so r < r and therefore the red rod's moment of inertia is lower.

- In a sentence or two, summarize what you found when calculating the torque on the lever arm balance.
- Briefly define *equilibrium*. How do the left and right torques relate to the condition for equilibrium?
- Briefly discuss some sources of error when balancing the lever arm.

Friction in pivot of lever arm balance is present, but very minor (the brass shaft rotates very freely); estimating when the apparatus is level is a bit tricky, as is measuring the value of *r* (which seems to produce a larger error on Loop 1 than the other two loops).