Vector Addition

Fall 2023

Introduction

In this experiment you will use a *force table* to learn how vectors are used to represent forces, and practice adding vectors algebraically and graphically.

Theory

Forces are vector quantities, described by both their magnitude and direction. We will use masses suspended from strings to exert force on an object – a metal ring. The *resultant* is a single force calculated as the vector sum of the forces exerted on the object. We will be able to calculate the magnitude and direction of the resultant vector on the force table by examining the *equilibrant*, which is a single force that establishes equilibrium by balancing two or more forces. *So, the equilibrant has the same magnitude, but opposite direction as the resultant.*

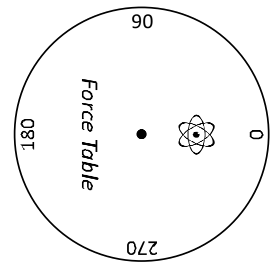

Figure 1

Before starting, make sure you are looking at your force table with 0° to the right, as shown in **Figure 1** (note that there are two angle circles on the force table; you'll be using the <u>*inner*</u> scale around the edge), and that the bubble level shows that the force table is level.

Addition of Two Vectors

You will add two vectors $\left(\vec{F}_1, \vec{F}_2\right)$ and calculate the magnitude and direction of the resultant $\left(\vec{F}\right)$ and equilibrant $\left(\vec{F}_{eq}\right)$, both algebraically and graphically. The forces will first be measured on the force table.

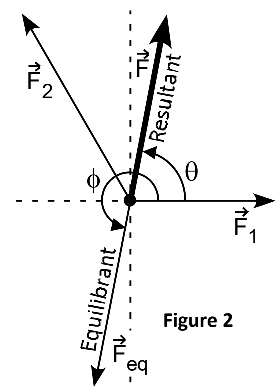

Figure 2

<u>Vector Addition Measured on Force Table</u>:

You will set up the vectors to be added on the force table (Figure 2): $\vec{F}_1$ is 100*g* at 0°, and $\vec{F}_2$ is 150*g* at 120°. Note that the units of "force" are left in *grams*; this will simplify your calculations and measurements. <u>It is not necessary to convert to Newtons!</u>

1. Attach two pulleys to represent the forces to be added, $\vec{F}_1$ and $\vec{F}_2$ with their direction as indicated in **Figure 2**. The third pulley will represent the *equilibrant*, used to balance the other two forces, so its position can be set approximately for now. *Do not over tighten the pulley clamps!*
2. Place the ring with three strings over the center post and pass each string over a pulley. Attach a mass hanger to each string, and place additional mass on the two strings that will represent the magnitudes of $\vec{F}_1$ and $\vec{F}_2$ (note that the hangers have a mass of 50 *g*).
3. Estimate the angle ϕ as follows: Grab the third string (representing the equilibrant), and gently pull the string while moving it left and right with respect to the table. Do so until the ring is centered on the post. Set the third pulley to the position you determined and hang the string representing the equilibrant over.
4. Now place additional mass on the third string representing the equilibrant. Recall that this force will balance the resultant force of $\vec{F}_1$ and $\vec{F}_2$. Adjust ϕ and the amount of force as necessary, making sure the strings pass straight over each pulley. Be sure to use the inner circle for your angle measurements.

5. The system is balanced (in a state of *static equilibrium*) when the ring is centered on the central post. Record your measurements of force and angle ϕ in a *Measurement Table* in your journal when the system is balanced.

Measurement Table

Force	Magnitude (g)	ϕ (°)	$\theta = \phi - 180°$ (°)
$\vec{F}_{eq}$			

6. Calculate the 'measured' value of $\theta = \phi - 180°$ and record in the *Measurement Table* above.

Vector Addition by Calculation:

7. Use trigonometry to calculate the magnitude and direction, θ, of the resultant vector, $\vec{F}$. You may find that using the *Calculation Table* below helps keep track of your calculated values; calculate F_x and F_y for each vector, then calculate the sum of the force along each axis. Use these sums to calculate the magnitude and direction of the resultant vector.
8. Calculate the angle $\phi = \theta + 180°$, which will be the angle of the equilibrant as measured on the force table (make sure you understand *why* you add 180° to θ to get ϕ) and record its value in a *Calculation Table* in your journal.

Calculation Table

Force	Magnitude (*g*)	Direction, θ (°)	F_x (*g*)	F_y (*g*)
$\vec{F}_1$	100	0		
$\vec{F}_2$	150	120		
			Sum F_x =	Sum F_y =
$\vec{F}$			$\phi = \theta + 180°$ =	

Graphical Addition of Vectors:

9. You will now add the vectors graphically. Use the intersection of two darker lines near the lower left corner of a new sheet of graph paper as the origin. Using a scale of 1 *cm* = 10 *g*, draw $\vec{F}_1$ in its direction and scaled length, beginning at the origin. You will find it easier to draw your vectors as shown in **Figure 3** so that the arrowhead does not obscure the end of the vector:

Figure 3

10. Now draw $\vec{F}_2$, scaled to the correct magnitude and direction, starting at the end of $\vec{F}_1$. Be sure to position the protractor correctly, as shown in **Figure 4**.

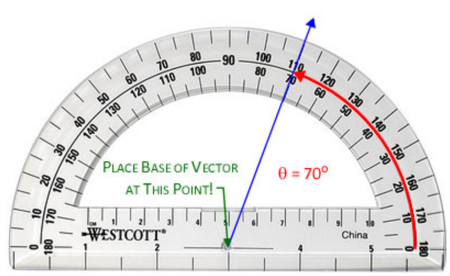

Figure 4

11. Draw the resultant vector, $\vec{F}$. Measure its magnitude with a ruler (in *cm*), converting back to units of grams. Measure its angle, θ, with a protractor, again making sure it is positioned correctly.
    - *Write the dimensions (in cm **and** grams) of your vectors on the graph you have drawn.*
    - *Label the angles that you measured.*
    - *Indicate the scale used on your vector map.*

Discussion

- Calculate and record in your journal the % difference between the following:
  - The magnitude of the force measured on the table and its calculated value.
  - The magnitude of the force measured on the table and measured on the graph.
    - *If you find either difference to be more than a few percent, you should check your force table measurements, calculations, and the lengths of drawn vectors!*
- Enter your results into your journal using the three methods in a *Summary Table*, as shown below.

Summary Table

	Measured on Table	Calculated	Measured on Graph
F (g)			
ϕ°			– N/A –
θ°			

- Which method do you think will calculate the magnitude and direction of $\vec{F}$ with the most precision? Briefly explain why and discuss the sources of error in the other two methods.

- Why do you think the smallest mass increment provided with the force table is 5-*g*? Why did you not get a pile of 1-*g* masses?

WHEN YOU ARE FINISHED, PLEASE:

- Remove clamps from the force table
- Untangle strings
- ***Return masses to their boxes (quantities inside box cover)***
- Place bubble level on top of the force table
- Arrange all parts neatly on the bench next to the force table