

Waves on a String

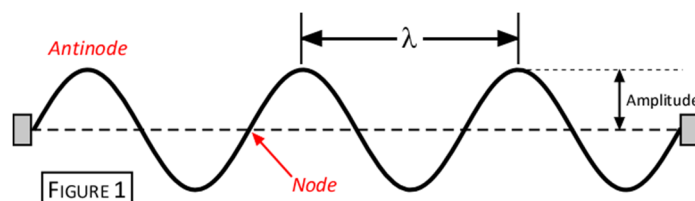
Fall 2023

Introduction

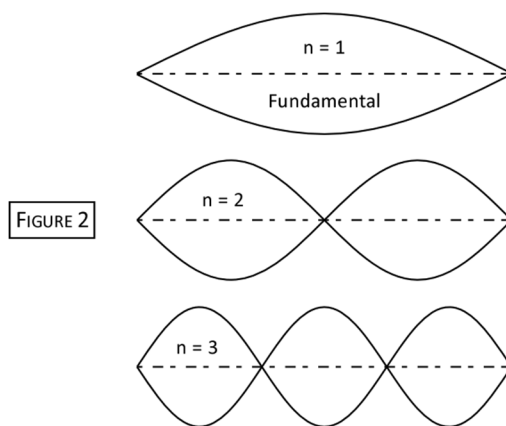
In this experiment, you will examine the properties of waves on strings by calculating the wave speed from the frequency and wavelength. The wave speed can also be calculated by examining the physical properties of the string used.

Theory

Standing waves are created by the superposition of waves traveling in opposite directions, left and right in our case. Those waves on a string clamped at both ends appear to stand still if half their *wavelength* just fits the length of the string between the clamps (FIGURE 1).



The *wavelength*, λ is the *distance* required for a wave to repeat itself, and the period is the *time* for a wave to repeat itself. *Frequency*, f is inversely related to the period and measured in units of *Hertz* ($\text{Hz} = \frac{1}{\text{seconds}}$). The *fundamental frequency* is the lowest frequency in which the wave repeats itself, with only one antinode, n (FIGURE 2).



The *wave speed*, v is calculated from the frequency and wavelength:

$$v = f \cdot \lambda \quad (\text{EQN. 1})$$

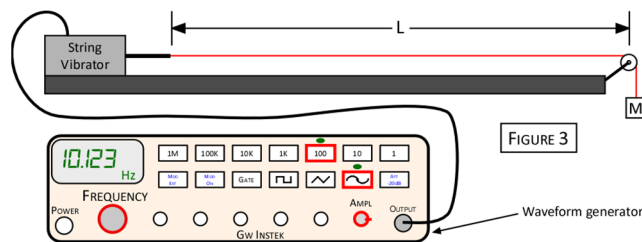
Applying Newton's law to a string gives a theoretical connection between wave speed, the tension in the string, T and the *linear density*, μ of the string:

$$v = \sqrt{\frac{T}{\mu}} \quad (\text{EQN. 2})$$

This week you will test these ideas by using a mechanical driver to vibrate a string at a variable frequency.

Experiment

The apparatus (FIGURE 3) consists of a waveform generator that provides a variable frequency wave to a mechanical vibrator. The vibrator is connected to the string, which causes it to oscillate. The tension in the string will be fixed by hanging a mass, M , at the end of the string, so that $T = M \cdot g$.



1. **Calculating Wavelength:** Carefully measure the length of the string, L from the end of the vibrating plate to the top of the pulley (FIGURE 3), and record in your journal. *Check the measured length with your instructor!*
2. Create a data table like the one below in your journal for $n = 2$ through 8, where n is the number of antinodes in the vibration. Calculate the wavelength, λ and record in the table:

$$\lambda = \frac{2L}{n}$$

3. Calculate and record $\frac{1}{\lambda}$ in the third column. This column will be used with the analysis and discussed later.
4. **Measuring Frequency:** Place a 500-g mass on the mass hanger (total mass with hanger: 550-g) at the end of the string. Turn the waveform generator on using the **POWER** button. Press the **100** and sine curve (\sim) buttons, as highlighted in FIGURE 3; a green light will appear above each button, and the units (Hz) will appear at the bottom right corner of the display. Check that the **AMPL** (amplitude) knob is turned to the 3 o'clock position as shown in FIGURE 3; you can increase to the maximum setting if needed.

Note: You will turn the **FREQUENCY** knob to change the frequency. When this knob is turned all the way off (counterclockwise), the display might read in the 900's, but you'll note that the units on the display are now in millihertz ($m Hz$) – which is lower than 1 Hz !

- a. Start by setting the frequency around 10 Hz . Increase the frequency by turning the knob clockwise *VERY SLOWLY*. Continue to increase the frequency until the string is resonating. You are looking for the frequency needed to make the string have the number of antinodes shown in column 1 of the data table.

Make sure that there is a node at the plate where the string attaches to the vibrator!

- b. Record the frequency displayed on the waveform generator for the number of antinodes in the data table (the frequency is displayed to 0.001 Hz ; you only need to record it to 0.1 Hz). Continue increasing the frequency for all the antinodes in the data table.
- c. Add a second 500-g mass to the mass hanger (total mass: 1050-g) and repeat your measurements.

| # of antinodes, n | Wavelength, λ (m) | $\frac{1}{\lambda}$ ($\frac{1}{m}$) | Frequency, f (550 g) (Hz) | Frequency, f (1050 g) (Hz) |
|---------------------|----------------------------------|---------------------------------------|------------------------------------|-------------------------------------|
| 2 | | | | |
| 3 | | | | |
| 4 | | | | |
| ... | | | | |
| 8 | | | | |

Analysis

1. Start Microsoft Excel and type in the last three columns of your data table, being sure to include the column titles. Create an X-Y scatter plot with $\frac{1}{\lambda}$ on the x-axis and *both frequencies* on the y-axis. *Keep the legend that Excel automatically adds to the bottom of your graph!*
2. Add a linear trendline for *each* data set. Remember to display the equation and increase the font size to 14 for each line!
3. Move the graph to a new sheet as follows: select the graph and click the **Move Chart** button in the *Design* tab of the *Chart Tools* group; select **New Sheet**, then **OK**. Make sure the graph has the correct title and axis labels; add your names to the header and then print a copy for each member of your group.
4. We know that the equation for a straight line takes the form $y = \text{slope}(x) + b$, and the wave speed is defined in EQN. 1.

Rearranging the definition of wave speed in the form that we plotted, we have:

$$f = v \left(\frac{1}{\lambda} \right)$$

Since f was plotted on the y-axis and $\frac{1}{\lambda}$ on the x-axis, the slope of the line therefore gives the wave speed, v .

5. Record the equation for each trendline as well as the wave speed for each string tension in your journal.
6. The physical characteristics of the string are needed for the analysis. First calculate the linear density of the string, μ , so that you can use it to calculate the values for the wave speed from the tension in the string. A simple method of calculating the linear density is to measure the mass of your string, m_{string} , and measure its length, ℓ_{string} ; dividing the two will give you μ .

$$\mu = \frac{m_{\text{string}}}{\ell_{\text{string}}}$$

It is difficult to disassemble the apparatus (please don't try), so you will determine the mass of an identical 'sample' string and measure its length to get a direct calculation of μ . Use the sample string found at the front of the lab, and calculate μ in units of kg/m .

7. Calculate the wave speed for each string tension using EQN. 2 and calculate the percent difference with the speed calculation from your Excel data.
8. Calculate the *fundamental frequency* for each string tension by solving EQN. 1 for f , using the longest wavelength supported by your string with one antinode ($\lambda = \frac{2L}{n}$, with $n = 1$) and the speed calculated in Step 7 for each string tension.

Make sure you use the length of your string, L – not the sample string length, ℓ_{string} !

Discussion

- Record your two values for the wave speed and their percent difference *for each string tension*. How well do the wave speeds for a given string tension agree with each other? Discuss what may have contributed to the difference between values.
- As you collected your data, what happened to the amplitude of the vibrations as you increased the frequency?
- How do the fundamental frequencies you found in Step 8 above relate to your frequency data? Can you see any rough pattern in your data related to this? *Hint*: Approximately how much does each frequency increase on your data table as you increase the number of nodes?