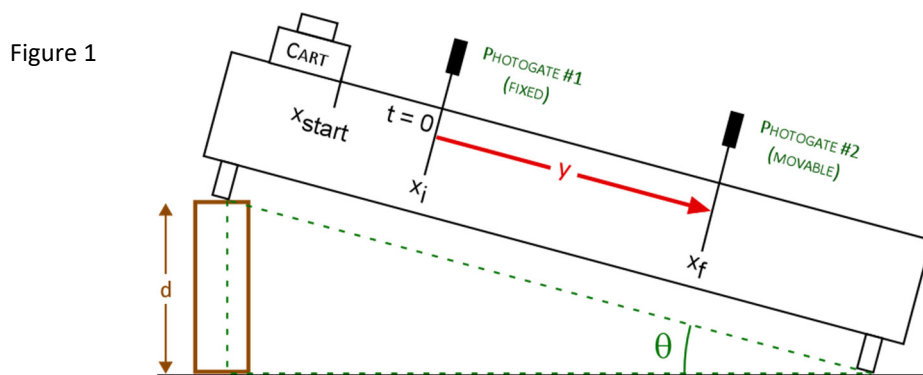# Kinematics: Constant Acceleration
Fall 2023

Purpose

Today we will measure the position of an air cart on an inclined air track as a function of time, determine its initial position, velocity, and acceleration from the data, and compare these to their expected values.

Theory:

Galileo first used an inclined plane to 'dilute' the motion of free-fall to reduce the acceleration, g, of a freely falling body to a value that was easily measurable. The model for the motion of a uniformly accelerating body is expressed as, $y = y_o + v_{oy}t + \frac{1}{2}a_yt^2$. The expected acceleration on an inclined plane is $a_y = g\sin\theta$, where θ is the angle of elevation of the plane. Galileo used his own sense of rhythm to mark time, and manually marked the position of a rolling ball at the end of each beat. You will measure time using a pair of photogates.

Figure 1

Procedure

I. *Track Setup*

1. Turn on the air supply and check that your air track is level so that a cart at rest remains so.

2. Turn off the air supply, then tilt the track by placing a block under the single support leg (<u>*Silver tracks*: make sure the double-legs are against the clamped board</u>). *Very carefully* measure the quantities you need to calculate θ, the angle of the track elevation from the horizontal. Write these measurements in a large, clear sketch of the tilted track, then calculate θ. Record the letter of the track used in your sketch. Show your instructor your calculated value of θ since this is often a source of error.

3. You will position the photogates so that they are tripped when the front edge of the cart is at a specific position on the track using the same procedure as you did in the *Conservation of Energy* experiment (refer to those instructions if needed):

    - Set the photogate control on the timer to the "pulse" setting, memory *on*; this will measure the time for the cart to pass between the two photogates.

    - Place the front edge of the cart at $x_i = 40.0$ *cm* and move Photogate #1 (*Figure 1*) until the <u>cart flag</u> makes the red LED on the gate light up. This will be the position of the cart at $t = 0$. Secure this photogate so that it cannot be moved during your experiment. <u>Recall that the tracks are marked in units of *mm*</u>.

    - Photogate #2 is aligned in the same manner, and it will be moved during the experiment. This photogate sits at position x_f, a distance $y = x_f - x_i$ downhill from Photogate #1. <u>You will start with the longest value of y possible on your track</u>.

4. *The cart will always be released* from the 25.0 *cm* mark on the track (x_{start} in *Figure* 1) for every trial. This means that the cart *does not* begin with a velocity of $v_o = 0$ at $t = 0$.

5. Set up a data table in your journal (start at the top of a new page). You will need columns for x_i, x_f, and y, a column with room for several time measurements for each y, and a column for the average of the times $\langle t \rangle$.

II. *Measurement of t and y*

6. Create a graph of y vs. t in Kaleidagraph; start with the largest possible y and measure t. Find the average time and plot the point.

    - Remember that you should start plotting your points *immediately*, as soon as they are calculated, so that you can see that the data follows some smooth function and appears as expected. *You can apply a fit to the data only after you have plotted three points!*

7. Add an entry for the origin {0,0} in your data table and plot this point; this will ensure that you have a better fit for your data.

8. Continue measuring t for other values of y, using your graph to choose values of y to fill the gaps; 6 or 8 points will do. ***Be sure to use the full length of the track to collect your data***.

9. Adjust the plot axes to show the origin (from the *Plot* menu, choose *Axis Options…*, then set *Minimum* to 0 on the *X* and *Y* tabs).

10. Before the air supply is turned off, measure the *expected* v_{oy} using the same procedure as in the two previous air track experiments (how did you *experimentally* measure the velocity?) Can you think of a reason why this might not be the best way to measure velocity in today's experiment?

Analysis

11. Select your KaleidaGraph plot and choose *Curve Fit → General → 2nd Order Poly w/Uncertainties*. This will fit a second order polynomial of the form $y = A + Bx + Cx^2$ to your data.

12. Create a second graph of the *residuals vs. time* from your data. Be sure to print both graphs.

13. Compare the fit function to the model for uniform acceleration and determine the corresponding physical quantity for each fit parameter (A, B and C). Determine the experimental equation of motion for your cart from the model, and record in your journal along with the SSR value.

    - A note about uncertainty: Today, $C = \frac{1}{2} a_y$, so $a_y = 2C$: this means that the value of C as well as the uncertainty in C are both doubled to get the acceleration and its uncertainty. Therefore, you will multiply the standard error for C in KaleidaGraph by *four* to get the uncertainty in a_y.

14. If there are no forces except the weight of the cart and the normal to the track, Newton's law predicts that the acceleration is constant, and the expected acceleration is $a_{expected} = g \sin\theta$. Calculate $a_{expected}$ from θ.

15. What is the expected value for y_o? (Hint: What is the value of y at $t = 0$?)

16. You calculated the expected value of v_{oy} in step 10 above. Calculate the % Difference between this expected velocity and the measured value (from KaleidaGraph).

17. Let us calculate the starting position on the track where the cart velocity was zero and compare it to its actual position.

    a) First, **algebraically** solve the kinematic expression $v_y = v_{oy} + a_y t_{start}$ for t_{start}, the time when the cart had a velocity of zero. Note that $v_y = 0$ here. What do you expect to get for the sign of t_{start}?

    b) Next, use your measured values of v_{oy} and a_y from KaleidaGraph to calculate t_{start}.

    c) Calculate the value of y at t_{start} using your experimental equation (*from step 13; don't worry about the parameter uncertainties*).

d) Finally, think carefully about this: *what distance, y did you just calculate*? Compare your calculated value with the actual distance used on the track. How well did they agree?

Discussion

- Create a summary table of your experimental results and their uncertainties, as well as your expected values for initial position, initial velocity, and acceleration. Remember to record your results that include uncertainties in the form $v_{oy} = \_\_\_\_ \pm \_\_\_\_$ {*units*}, etc.

- Restate the track that you used, its measured angle, and the experimental equation of motion and SSR value.

- Create number lines to represent your experimental results. Discuss the agreement or discrepancy between the value of these parameters and their expected values. Are the theory and data consistent? If not, explain why. Be sure to consider the method used to determine each of your values.

- Include a brief discussion of measurement uncertainty and experimental error.

- Briefly discuss your residuals plot (were the residual points randomly scattered, or seem to follow some pattern?)

- Finally, state and discuss your results for *y* at t_{start}, <u>recalling that this was when the cart had zero velocity</u>. What value did you expect to find for this quantity?

PLEASE TURN OFF THE PHOTOGATES AND REMOVE ANY TAPE USED TO SECURE THEM. RETURN THE TRACK TO ITS LEVEL POSITION ON THE BENCH.