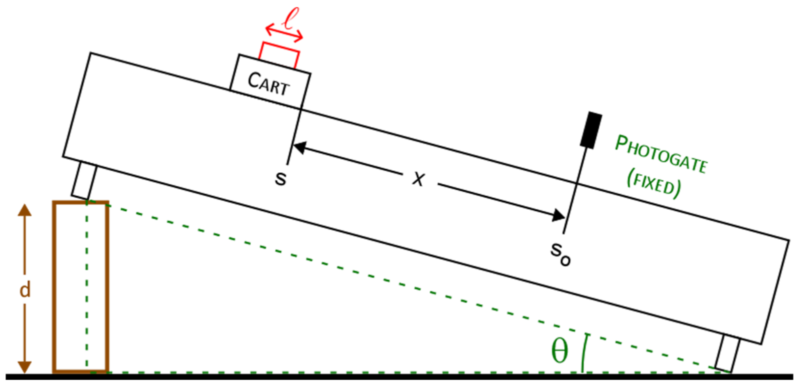
Conservation of Energy on an Inclined Plane

Fall 2023

Purpose

Today you will perform an experiment to show that the final kinetic energy of an object sliding down a frictionless incline is proportional to the initial potential energy.

Figure 1

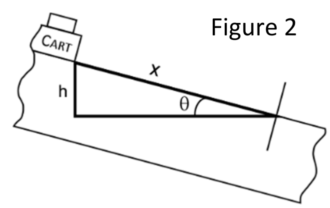


Theory

If a cart is initially at rest, it falls *vertically* a distance h as it travels a distance x down the track (*Figure 2*). Energy conservation tells us that the final kinetic energy ($K_f = ½mv^2$) equals the initial potential energy ($U_i = mgh$). Therefore:

$$K_f = U_i$$
$$K_f = mgh$$
$$\frac{K_f}{m} = gh \qquad \{Eqn.\ 1\}$$

Figure 2



In this experiment, the cart is sliding without friction down an incline. But since the interaction between the track and glider is always perpendicular to the cart's velocity, we can ignore this force and the equation above still holds. *Figure 2* shows that as the cart travels a distance x along the track, the vertical distance it travels is $h = x \cdot \sin\theta$. Substituting the expressions for h and K_f into *Eqn. 1* above gives:

$$\frac{K_f}{m} = g \cdot x \sin\theta$$
$$\frac{½ mv^2}{m} = g \cdot x \sin\theta$$
$$\frac{v^2}{2} = g\sin\theta \cdot x \qquad \{Eqn.\ 2\}$$
$$y = slope \cdot x + b$$

The result above shows that $v^2/2 = K_f/m$, <u>so a measurement of velocity squared is proportional to a measurement of kinetic energy.</u> Likewise, since $h = x \cdot \sin\theta$, and $U_i = mgh$, <u>x is proportional to a measurement of potential energy</u>. As the derivation of *Eqn. 2* shows, plotting $v^2/2$ vs. x should produce a linear graph with slope equal to $g \cdot \sin\theta$ and the y-intercept through the origin (review <u>Analyzing the Graph</u> from the *Fitting Data to a Mathematical Function* instructions if you're not sure why this is true.)

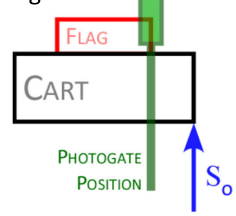
Procedure

I. *Track Setup*

1. Look <u>carefully</u> at *Figure 1* and then take measurements that will allow you to calculate θ, the angle of the track elevation from the horizontal; *note that some of these measurements will be easier to perform while the track is level on the bench!* Record your measurements on your sketch of the tilted track (which should be drawn **<u>clear</u>** and **<u>large</u>**), then show your calculation of θ (in degrees.) Your sketch should also include the track letter and cart number used. <u>Check your angle with your instructor</u>; this calculation is often a source of error.

2. Turn the air supply on while the track is not elevated, and check that the track is level by gently placing a cart in the center of the track. Ask your instructor for assistance if the cart has significant motion. Turn off the air supply, then tilt the track by placing a block under the *single* support leg (<u>*Silver tracks*: make sure the double-legs are against the clamped board</u>).

Figure 3



3. It is easier to place the photogate timer in position *before* turning on the air supply. Hold the cart with its leading edge at the 160.0 *cm* mark of the tilted track (<u>note that the tracks are marked in units of *mm*</u>). Turn the timer on in "gate" mode (<u>keep the memory *ON* so you don't need to catch the cart</u>) and move it along the track until the red LED on top of the gate lights; this indicates that the timer will be triggered by the leading edge of the flag (*Figure 3*). This defines the coordinate of s_o.

4. Tape the timer to the bench so that it will not move during your experiment. Measure ℓ, the length of the flag using a vernier caliper, following the instructions in the document "*Reading a Vernier Caliper*".

II. *Measurement and Graphing*

5. Set up a data table with the following headers (you will be collecting <u>a lot</u> of data, so start at the *top* of a new page):

s_o (cm)	s (cm)	$x = s_o - s$ (cm)	t (s)	$\langle t \rangle$ (s)	v (cm/s)	$\frac{v^2}{2}$ $(cm/s)^2$

6. Move the cart up the track to position $s = 25$ *cm*; this position gives the longest distance x. Hold the cart in place with the tip of your pencil eraser, then pull your pencil straight out to release the cart. The photogate will measure the time it takes the flag to pass through the timing gate.

7. Repeat (and record) the measurements with the same x; three or four measurements should give you reliable data. Draw a light line through time measurements you think are wrong. Calculate the average time, $\langle t \rangle$, the velocity (*think carefully about this calculation!*) and $v^2/2$ from the average time, and then plot the point on a KaleidaGraph plot of $v^2/2$ vs. x. *It is very important that you plot your points in Kaleidagraph as they are calculated!*

> Select the KaleidaGraph plot and display the origin of the axes: from the **Plot** menu, choose **Axis Options**. Change the *Minimum* value to 0, click the 'Y' tab at the top of the dialog box, again set the *Minimum* to 0, and click OK. This will ensure that you take measurements along the entire length of the track, not just at one end. Also note that Kaleidagraph requires that <u>at least 3 points</u> are plotted before you can fit a function to the data. Ask your instructor if you would like to use superscripts in your axis labels.

8. Set x to 10.0 *cm*, the smallest distance you'll use, and measure t. Repeat for other values of x, using your graph to choose values of x to fill the gaps; at least 6 distances will give good results. <u>Be sure to spread your points out over the *entire* length of the track</u>.

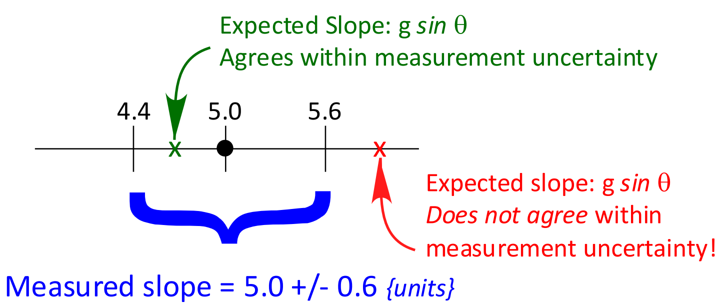
Analysis

9. Look at your completed graph. What function will fit the data best? (*Hint*: refer to the *Theory* section.) Use KaleidaGraph to fit a function to your data (but don't use "Linear through origin"!) Follow the instructions in "Graphing & Curve Analysis using KaleidaGraph" to calculate the slope and intercept. Be sure to include the uncertainty with your results (use *twice* the calculated standard error).

10. Spend some time thinking about what the model predicts you should find for the values of the slope and intercept. Do your results agree with your model's prediction? If not (and there is a strong possibility that you might find a discrepancy), check to see if your results agree with other groups. What will this tell you?

Discussion

- Write down the numerical results of your analysis (*including the uncertainty and SSR*) and a reference to the graph.
- Present your slope *and* intercept values, along with the uncertainty for each (and the units!) Recall that the uncertainty for each parameter is *twice* the standard error. For example, if your slope is 5.0 *{units}* and the standard error is 0.3 *{units}*, then your slope is 5.0 ± 0.6 *{units}*. This means that the *measured slope* is between 4.4 and 5.6 *{units}*; of course, you will use the correct values and units for your slope. Present your slope results using a number line, as shown below:



- Discuss whether the slope calculated using KaleidaGraph (*the experiment*) is equal to the value you expect (*the theory*). Be sure to consider the uncertainty in your slope. In the number line above, you can clearly see that one value of the expected slope falls within the range of uncertainty for the measured slope, while the other falls outside the range of uncertainty.
- Discuss how the theory of energy conservation was used to determine your results. Also explain what the quantities $v^2/2$ and x represent on your graph, and why the theory predicts that the slope of this graph should be $g \cdot sin\theta$ (*Hint*: If you think $v^2/2$ means "half the velocity squared", you need to reread the **Theory** section)
- You may get some unexpected results in this experiment; consider the following:
  - What is the expected value of the y-intercept? Do your expected and measured values of the intercept agree with each other? Why or why not? *Hint*: Think about what the expected value of the y-intercept *means*. What happens with the cart's motion at the end of its run?
  - The track inclination, θ, is a relatively small angle, so it is sensitive to small errors. Calculate what the value of θ would have to be to make $g \cdot sin\theta$ equal to the slope on your graph. Compare this value of θ with your measured value, and comment on the differences and its effect on the results. Other than measurement uncertainty, what might be affecting your calculation of θ? *Hint*: What assumption are we making about the lab bench in step 2?

PLEASE TURN OFF THE PHOTOGATE AND REMOVE ANY TAPE USED TO SECURE IT. RETURN THE TRACK TO ITS LEVEL POSITION ON THE BENCH.