

## Kepler's 2<sup>nd</sup> Law

Fall 2021

### Introduction

In this experiment, you will examine the motion of an air puck traveling in an elliptical path. Applying the theory of conservation of angular momentum, you will see if the puck moves as predicted by Kepler's 2<sup>nd</sup> Law.

### Theory

The experimental apparatus consists of a table with a sheet of carbon paper and newsprint on top. A 550 g air puck is attached to the table by four springs (Fig. 1), and connected to a spark-timer and an air supply. The puck rides on a cushion of air so that there is very little friction between it and the newsprint. The spark-timer records the position of the puck on the newsprint at specific intervals.

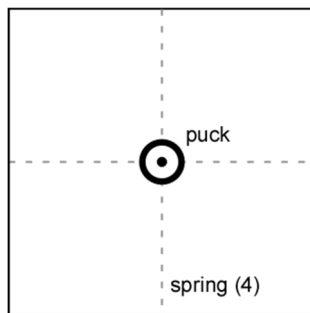


Fig. 1: Puck at rest

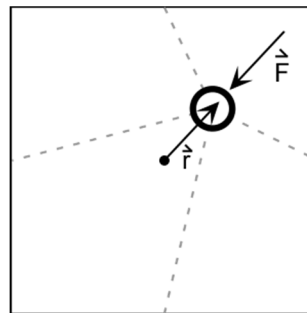


Fig. 2: Puck in motion

When the puck is displaced a distance  $\vec{r}$  from its equilibrium position, it experiences a restoring force,  $\vec{F}$  created by the springs (Fig. 2). If  $\vec{F} \parallel \vec{r}$ , then the torque  $\vec{\tau} = \vec{r} \times \vec{F} = 0$ , and the angular momentum,  $\vec{L}$  is constant. Constant angular momentum implies Kepler's 2<sup>nd</sup> Law: an object traveling in an orbit will sweep out equal areas in equal intervals of time (Fig. 3). For this to be true, the *speed* of the object must change as it travels along its path.

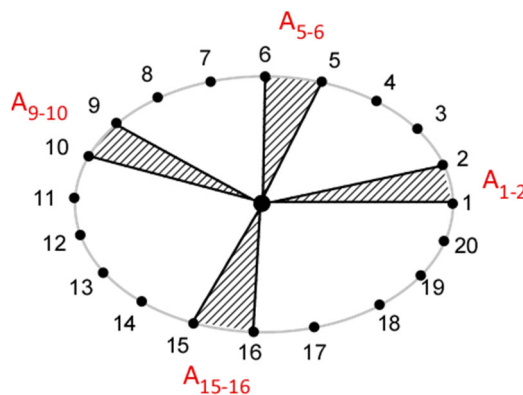


Fig. 3: The shaded areas are equal

## Experiment

1. Your instructor will assist you in the operation of the air table. Turn on the air supply and practice releasing the puck in a circular motion. Keep the size of the orbit about that of a standard letter-size sheet of paper; this will keep your areas more consistent with each other. Be very careful that you do not tilt or push down on the puck, as the carbon paper underneath is very easily torn; holes in the carbon paper will inhibit the puck's frictionless motion.
2. The spark timer is set to **StandBy** and the rate to 15 sparks per second. With the puck at rest, have your partner quickly turn the knob from **StandBy** to **Spark On**, and back to **StandBy** again. This will mark the paper at the center of the pucks orbit.

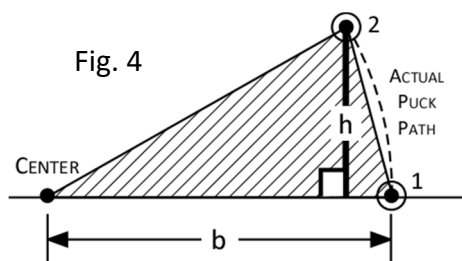
**CAUTION:** Keep your hands off the edges of the air table and the puck while the spark timer is on. You might get a shock from the apparatus.

3. With the spark *off*, start the puck moving in a circular path. Your partner will again turn on the spark timer, then turn it off when the puck has made one complete orbit.
4. Lightly mark the paper with an arrow indicating the orbital direction. Remove the puck and the sheet of newsprint; the orbital path will be marked on the underside of the sheet. *Indicate the starting and end points*, the direction of motion, and the setting of the spark timer on the paper.

**Note:** It is very important that  $t = 1$  spark is marked as the *first* point!

## Analysis

5. Carefully circle and number every point on the newsprint. Stop if the points spiral in and overlap.
6. Use a ruler to draw a line connecting each point to the center of the orbit. Then draw a line connecting *every other pair* of adjacent points (e.g. between point 1 and point 2, then between point 3 and point 4, etc.) as shown in Fig. 4.
7. Finally, for the point pairs that were connected, draw a line that starts at one point, and intersects the opposite line *perpendicularly*, as shown in Fig. 4. This line will represent the height of each triangle.
8. Measure the base,  $b$  and height,  $h$  for each triangle drawn. Use these measurements to calculate the *area* swept out in a particular time interval, and record in a table (estimate your measurements to 0.1 cm):



Time Interval	Sparks	$b$ (cm)	$h$ (cm)	$A = \frac{1}{2}bh$ (cm <sup>2</sup> )
1	1 – 2	—	—	—
2	3 – 4	—	—	—

9. If angular momentum is conserved, then the areas of these triangles should be the same. Use KaleidaGraph to create a plot of area vs. the time interval. What would you expect this graph to look like if angular momentum is conserved? Place a linear fit on the graph to see if there's a noticeable trend (*the line equation isn't important here, so it can be hidden*). Adjust the scale of the vertical axis so that it begins with zero, and then print the finished graph.
10. Create another KaleidaGraph plot of the residuals vs. the time interval (see "Graphing & Curve Analysis with KaleidaGraph"). A pattern in the residuals might indicate that a systematic error occurred during the experiment. Do you notice any pattern?
  - This graph is easier to analyze if you 'zoom out' from the points: set the range of the vertical axis from +1 to -1, or +2 to -2 (depending on your data). Also, choose **Smooth** from the **Curve Fit** menu to help determine if a pattern is present. Print this residuals graph.

**Project (Optional, +1 point):**

- Carefully align the transparency grid *along the major axis* of the ellipse (use 2 grids if the orbit is too large), and then find the coordinates of each point. Tabulate your results as shown below (use *gu* – grid units – for the units). (*Note:* It's important that you take care when aligning the grid along the major axis!)

$t$ (sparks)	$x$ (gu)	$y$ (gu)
1	—	—
2	—	—

- Plot  $y$  vs.  $x$  in KaleidaGraph to check for bad coordinates, and then print the graph. The graph should look like your original spark sheet.
- The equation for an ellipse is shown below, where  $A$  is the semi-major axis, and  $B$  is the semi-minor axis (Fig. 5). Use KaleidaGraph to see if the orbit is *really* an ellipse:

$$\left(\frac{x}{A}\right)^2 + \left(\frac{y}{B}\right)^2 = 1$$

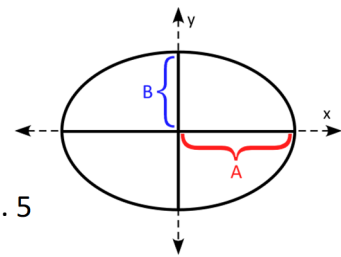


Fig. 5

*Hint:* Plot  $y^2$  vs.  $x^2$ . Algebraically rearrange the equation above so that you solve for  $y^2$ . How does this rearranged equation tell you what this graph *should* look like? Be sure to print this graph as well.

*Note:* Plotting  $y$  vs.  $x$  and saying it *looks* like an ellipse is not a valid demonstration! Keep in mind that *the values 'a' and 'b' from KaleidaGraph are not the semi-major (A) and semi-minor (B) axes of the ellipse!*

- How do the values of  $A$  and  $B$  measured directly from your ellipse compare to those calculated from the graph you just plotted?

**Discussion**

- Was angular momentum conserved? What did your graph of area vs. the time interval show? By what percentage did the triangle area decrease (from the largest to smallest area)? If angular momentum was not conserved, explain what you think would account for this deviation from expected behavior?

**ATTACH THE "TARGET" SHEET CONTAINING YOUR ELLIPSE TO ONE OF YOUR LAB REPORTS. USE SCISSORS (IF AVAILABLE) TO TRIM DOWN THE SIZE OF THE TARGET SHEET.**