# Creating Graphs with Mathematica

Fall 2005

## Introduction

Mathematica is a powerful symbolic computer language. This exercise is designed to introduce you to Mathematica's graphing capabilities in the context of the class work we are doing in special relativity. *Each person will perform this exercise by themselves*.

### *Starting Mathematica*

When using a computer in an SLU Physics lab, double-click the *Mathematica* icon on the desktop. For other computers on campus, you'll find Mathematica in the SLU Network menu, in the "Software for Courses", "Physics" folders.

### *Issuing Mathematica commands*

1. The basic Mathematica notebook is a document that contains text, input, and output. To enter an input statement you simply start typing, and press the Enter key to activate the statement.

   > *Note:* There are two *Enter* keys on the keyboard, and Mathematica treats them differently. The *Enter* key that you normally use when typing (located above the Shift key) will put a carriage return in a statement line. The other *Enter* key, located on the number keypad, will activate the statement line. Get in the habit of using the *Enter* key on the number keypad for Mathematica workbooks!

   Here is an example of a Mathematica statement that assigns a value to a variable. Type **m:=2** and press the Enter key. You'll see the following in the notebook:

   ```
   In[1]:=    m:=2
   ```

   *Always use* **:=** *for the assign statement, with no spaces between ":" and "="*. The notation `In[1]:=` shows that this statement is an Input statement, and it is line number 1 (this line number might change as you edit your notebook). *`In[1]` will appear automatically in the notebook; do not type it. Only type the commands that appear in boldface in these instructions.* Note that variables are case sensitive, so 'm' and 'M' would be recognized as distinct variables.

   Type **m** and pressing Enter to evaluate this variable. You'll see the following:

   ```
   In[2]:=    m

   Out[2]=    2
   ```

   The notation `Out[2]:=` shows that this line is an output statement, containing the results of the evaluation.

2. Mathematica can also be used as a calculator; try the following, and see if your output matches:

   ```
   In[3]:=    m^2 + m^3

   Out[3]=    12
   ```

3. Let's define a function. Use the example of γ in special relativity (Greek letters are generated by pressing the <ESC> key before and after a letter, e.g. **<ESC> g <ESC>** will produce the greek letter 'gamma'):

```
In[4]:=    γ:=1/Sqrt[1-v^2]
```

Note that the command for 'square root' begins with an uppercase 'S'. Mathematica commands are case sensitive as well. Evaluating gamma should produce the following output:

```
In[5]:=    γ
```

Out[5]= $\frac{1}{\sqrt{1-v^2}}$

4. To plot γ as a function of *v* for the domain [0,1], enter the following, again noting the case of the Plot command:

```
In[6]:=    Plot[γ, {v, 0, 1}]
```

Note that arguments for the Plot command are enclosed by square brackets "[ ]", and the *variable ranges* are enclosed by curly brackets "{ }". The graph will appear below your function.

5. Graphs can be easily resized. Select the graph and drag its lower-right corner to stretch it.
6. Now let's see how you can restrict the *vertical* range of the function being plotted to lie between 0 and 5 (You can type the new line below, or edit the previous statement and re-execute it. Any statement can be re-executed by placing the cursor *anywhere* in the line and pressing the Enter key).

```
In[7]:=    Plot[γ, {v, 0, 1}, PlotRange -> {0, 5}]
```

Notice how γ is practically equal to 1 for speeds less than 0.2.

7. More than one function can be plotted on the same graph. Simply list the functions to be plotted within curly brackets. Try the following, which will plot three different functions:

```
In[8]:=    Plot[{v, v^1.5, v^2}, {v, 0, 2}]
```

Note that no labels appear on the graph; it's up to *you* to know which function is associated with a particular curve.

*Comparing classical and relativistic formulas for momentum and kinetic energy*

8. According to Newton, the equation for momentum is $p = mv$. In relativity it is modified (because time and length differ in different reference frames) to $p_r = \gamma mv$. Let's define these functions in Mathematica and plot them. We have already defined γ in step 3, and have already assigned the mass to be 2. It is hard to use units in Mathematica, so we will assume standard (SR) units. This means that you will have to be sure to keep track of the units for yourself.

```
In[9]:=    p:= m*v
```

Note that an asterisk is required between 'm' and 'v' to indicate multiplication. Evaluate the function you've just defined, and enter a function for the relativistic momentum:

```
In[10]:=    p

Out[10]=    2v

In[11]:=    pr:=γ*m*v

In[12]:=    pr
```

Out[12]= $\dfrac{2v}{\sqrt{1-v^2}}$

9. Plot the functions $p$ and $p_r$ *on the same graph* and see how they agree for small velocities (from $v = 0$ to 0.5).
10. Define a third function, $p_f$, for the fractional difference $p_f = (pr - p)/p$. Create a new plot with all three functions, setting the range of $v$ from 0 to 1 and the vertical range between 0 and 10.
11. The relativistic equation for kinetic energy is $K_r = (\gamma - 1)m$. Make a double function plot comparing to the classical expression for kinetic energy for $v = 0$ to 1. Change the range on the scales to see where $K$ and $K_r$ begin to diverge.
12. Use the instructions below to print the graphs from steps 10 (containing $p$, $p_r$, and $p_f$) and 11 ($K_r$ and $K$) on a *single* sheet of paper. Be sure to label the graphs, and briefly discuss the results.

*Printing Graphs*

- At the far right of every input and output statement you'll find a square bracket. Click this bracket in the output line containing the graph you wish to print (**<Shift-click>** to select contiguous lines, **<Ctrl-click>** to select non-contiguous lines). Choose **Print Selection** from the **File** menu. *If you don't follow this procedure, you'll print your entire notebook!*

## Discussion

- Hand in your graphs from steps 10 and 11. On the graph, label each function, and write a brief description of its significance.