Projectile Motion

Fall 2023

Introduction

The purpose of this experiment is to measure the initial velocity of a projectile, then calculate the distance it will travel when fired at an angle. A trash can placed at this distance will demonstrate your understanding of the theory.

Theory

We begin with an FBD for a projectile fired horizontally from a table, neglecting air resistance.

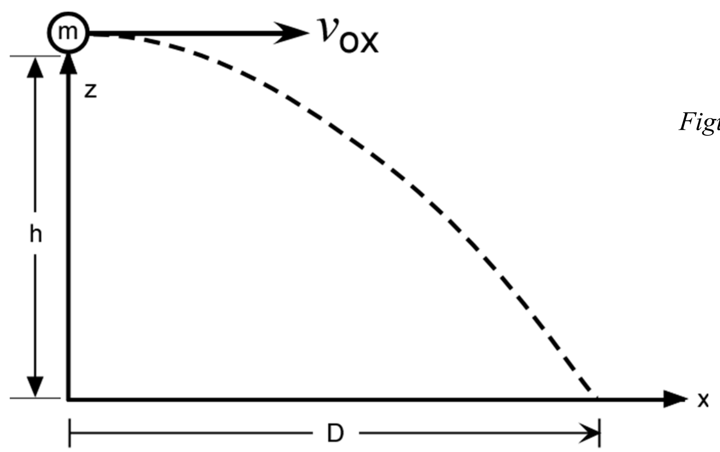
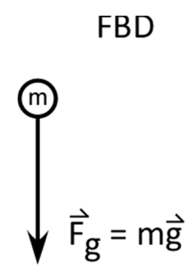


Figure 1



Since the force is constant, we can immediately write the following:

$$z = \tfrac{1}{2} a_z t^2 + v_{oz} t + z_o \qquad \text{(Eqn. 1)}$$

$$x = \tfrac{1}{2} a_x t^2 + v_{ox} t + x_o \qquad \text{(Eqn. 2)}$$

We see from the force diagram (*Figure 1*) that $a_x = 0$, and $a_z = -g$. We can use these two equations to determine how much time it takes for the projectile to hit the floor. Then, from *where* it hits ($x_f = D$), we can determine how fast it must have been going when it left the launcher, v_{ox}.

1. The coefficients in the polynomials (1) and (2) above are x_o, z_o, v_{ox}, v_{oz}, a_x, and a_z. We can use the symbols in the sketch for the initial positions and velocities, and from the FBD get the accelerations to rewrite these six coefficients for our projectile (three of them are zero!).
2. Now rewrite Eqns. (1) and (2) with the new coefficients and solve them for *t(g,h)*, the time to hit the floor when *x(g,h) = D*, the horizontal distance traveled before hitting (<u>it is not necessary to solve for *t* numerically</u>!)
3. Finally, derive an expression for v_{ox} *(g,h,D)* – the initial velocity of the projectile as a function of *g*, the height of the projectile, and the distance it travels. (Your final expression should contain *g*, *h,* and *D*, ***not*** their numerical values!) Check that your derived expression produces a result with units of *velocity*!

   Make sure that your derivation shows how you connected the six coefficients in Eqns. (1) and (2) to their respective values.

Experiment

Part I: Measuring the Initial Velocity

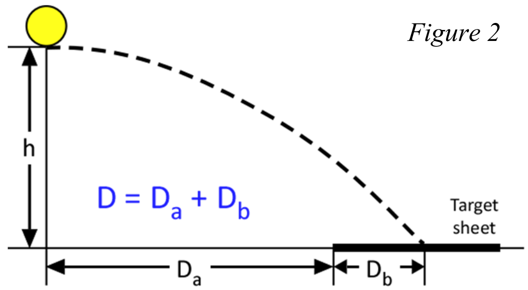


Figure 2

The Ballistic Method

1. Your instructor will show you how to load and shoot the spring launcher *horizontally* ($\theta = 0°$) from a tabletop. Measure the height, **h**, of the projectile above the floor (measure to the *bottom* of the picture of the projectile on the right side of the launcher). In your sketch, indicate the launcher you used (A, B, C, etc.)

   Note: Be sure to set the launcher for ***medium*** *range throughout the experiment; the long-range setting will hit the ceiling or opposite wall!*

2. Fire the launcher a few times, noting the place the projectile hits the ground. *Be sure to check that the launcher angle has not changed after each shot*. Place a wooden catch box at this location; fire a few more shots to check its position. Remove the catch box and replace it with a target sheet (ink-side-up carbon paper under a sheet of graph paper). Tape the graph paper to the floor and use another piece of tape to mark the front position of the target sheet; <u>don't remove this tape until you are ready to leave the lab so that you can recheck measurements if needed</u>.

3. Measure the distance along the floor from a position directly below the release point of the projectile (the side of the launcher is marked with a small cross) to the front edge of the target sheet. Call this distance $\mathbf{D_a}$.

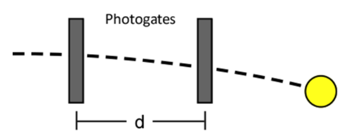
4. Fire the launcher *at least* 10 times. The projectile will leave a mark on the underside of the graph paper. After your 10 shots, pick up the target sheet and measure the distance from the front edge of the paper to the center of each spot, $\mathbf{D_b}$. (Don't worry about which is the first shot, the second, etc.)

5. Calculate the horizontal distance, $\mathbf{D} = D_a + D_b$ the projectile traveled for each shot (*Figure 2*). Also calculate the minimum, average, and the maximum distance; denote these as $\mathbf{D_{min}}$, **<D>**, and $\mathbf{D_{max}}$.

6. Use the equation you derived for the ballistic initial velocity to find the minimum, average, and maximum velocity of the projectile.

Measuring Time of Flight

Figure 3



7. Attach the double–photogate assembly to your launcher. Position it so that the first photogate is as close to the end of the launcher as possible. Set the timer to *pulse, memory on*.

8. Repeatedly fire the launcher through the pair of photogates, then calculate the initial velocity (*Figure 3*). Compare the average *ballistic* velocity with the average *time-of-flight* velocity and record the %difference. *Note*: You should check your measurements and calculations if you find more than a 5% difference between these average velocities.

9. Remove the photogate assembly from the launcher.

Part II: Sink the Ball!

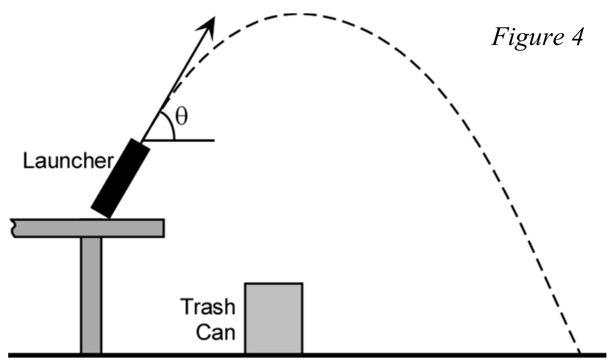


Figure 4

10. The algebra becomes a little messier when the projectile is shot at an angle, so you will use a Microsoft Excel spreadsheet to determine the distance your projectile will travel. Set your launcher to any angle between 30° and 50° (Figure 4 – *Note that the launchers in the hallway and next to the lab printer can only use an angle between 30° and 45°!*) The height of the projectile should be the same as before (*check it!*).

Open the Excel spreadsheet *Projectile Motion* in PHYS151 of the T: drive (*Click the **Enable Content** button if the security warning appears*). Enter your names and select the launcher used from the pull-down menu. Enter the projectile height, launcher angle, and your velocities from the ballistic method. Click the **Graph** button to see the trajectory showing where the projectile will hit the floor.

11. Click the ***Print This Graph*** button (this will add your names to the graph) and print a copy for each person in your group. You will use this graph to determine where you will place the can to catch the projectile when fired at an angle. You can (and probably should!) draw on the graph printout as needed. **Draw and label a sketch like *Figure 4* in your journal**, carefully place the can at the calculated position, and take your shot. The rules are as follows:

    a. The placement of the trash can is accomplished by calculation and measurement, *not* trial and error. Once the can is placed on the ground, it *cannot* be moved closer to, or further from the launcher; only left-right adjustments will be allowed. No practice shots are allowed; your instructor must witness your trash can shot!

    b. If you miss the can on the first attempt, you will be given a second shot, *without* moving the launcher or changing the distance to the can; left-right adjustments of the can will be allowed to account for aiming errors. Further attempts will not count. Rim shots count only if the projectile bounces into the can (which rarely happens). *You will be given credit for a first shot if you sink the ball in the can on the second shot after making only left-right adjustments!*

    c. You will earn +1 extra point on your lab if you sink the projectile into the can on the first shot.

    d. For the more daring amongst you, a smaller target will be made available (as time allows). You must choose a different launch angle if you want credit for this smaller target.

It is traditional to gather the whole lab section together to share in your success!

Discussion

- Record your numerical results: the three initial velocities, the photogate velocity, and the percent difference.
- Describe *how* you determined the placement of the trash can. Be sure to specify the distance where you placed the can. Do the same if you used a smaller target (a juice can) as well.
- If the ball missed the can on the first try, discuss the reasons why it missed. *Be sure to state if the shot fell short or went too far.*
- If you used a smaller target, report on what happened. Why is it more difficult to this the smaller target?
- Discuss the errors involved in this experiment. What may have affected the results?
- Attach the target sheet to the back of the lab journal of one member in your group.

BEFORE LEAVING THE LAB, PLEASE REMOVE ANY TAPE YOU APPLIED TO THE FLOOR OR THE LAB BENCH AND RETURN THE PROJECTILE TO THE BOX UP FRONT.