

The Speed of Light

Fall 20xx

Solutions

Introduction

There are two parts to this experiment: You will measure the speed of light using an apparatus set up in the hallway; then you will determine the speed of light in different reference frames using space-time diagrams.

Experiment A: Measuring the Speed of Light in the Hallway

The measurement of the speed of light is relatively straightforward. A schematic of the apparatus appears in Figure 1 below¹:

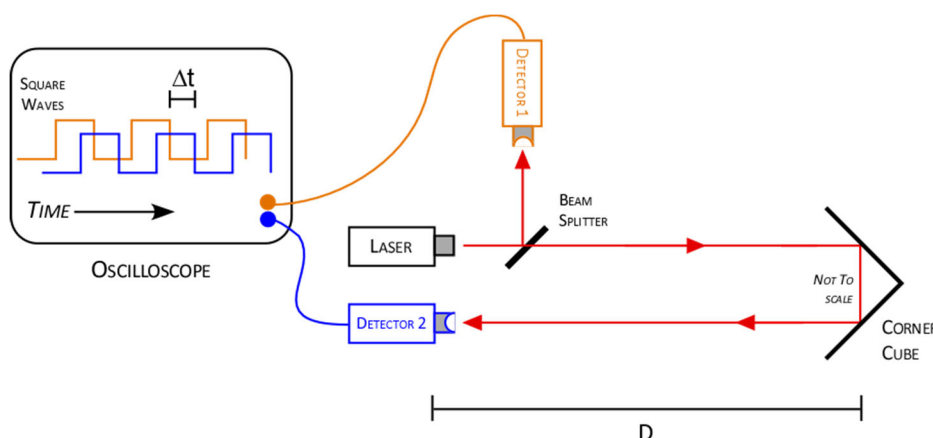


Figure 1 - Speed of light apparatus

A laser beam passes through a beam splitter, a semi-transparent mirror that both reflects and transmits the laser light. Part of the beam is directed at *Detector 1*, and a trace is produced on the oscilloscope (your instructor will show you which trace is created by this detector). The rest of the beam passes down the hallway until it hits the *corner cube* mirror. A corner cube uses three mirrors attached to each other at right-angles, so that an incident light ray striking these mirrors at any angle is reflected directly upon itself (Figure 2), as opposed to a plane mirror which reflects light at the same angle as the incident ray (Figure 3). Be sure to examine the optical properties of the sample corner cube in the lab. Note that the vertical distance of the laser beam in the corner cube of Figure 1 is insignificant.

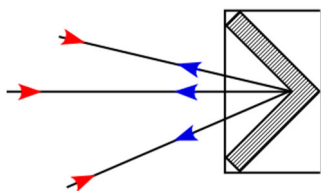


Figure 2 – Corner Cube

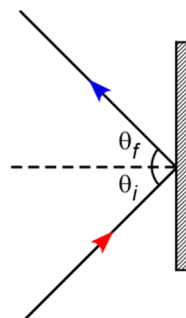


Figure 3 – Plane Mirror

The reflected ray leaves the corner cube and heads back toward *Detector 2*, causing a second trace to appear on the oscilloscope. This second trace occurs a time Δt after the *Detector 1* trace. If you know the distance the beam

¹ Apparatus design adapted from “The Speed of Light: Making an Easy Time of It”, Burt Brody, *The Physics Teacher*, Vol. 41, p. 276, May 2003

travels (from the laser to the corner cube then to *Detector 2*) and the time interval Δt , you can easily calculate the speed of light.

1. Your instructor will explain the operation of the laser and oscilloscope. A laptop duplicates the display of the oscilloscope.
2. Carefully measure the distance from the laser to the corner cube, and back to *Detector 2*. Note that *Detector 2* is actually located further away from the corner cube than the laser.
3. Align the cursors on the oscilloscope with the peak for each trace; note that Δt appears on the display. On the laptop, choose **File**, then **Print/Save** to print one copy of the screen for your group.
4. Calculate the speed of light (**in m/s**) from your distance and Δt measurement (be careful of your units!). Calculate the % difference between your measured and the expected value of 'c'.

Experiment B: Coordinate Transformations

In this exercise you will determine the speed of light (in SR units) by calculating the time it takes for a pulse of laser light to travel a known distance to a mirror and back. Next, you will set up the coordinates for the events for this measurement (in the Home Frame), and draw the worldline of the light pulse on a space-time diagram. Then you will transfer the events into a moving (Other) reference frame, first using the Galilean transformations and then the Lorentz transformations, make new space-time diagrams. Finally, you will draw some conclusions about the speed of light in the other frame.

I. Configuration of Experiment

1. Let the laser be located at the origin and the mirror at $x = 30 \text{ m}$ (Figure 4). You will want to convert to SR units right away. Define the measuring events like this:

Event A: Light pulse leaves laser at $t = 0$.

Event B: Pulse reaches mirror and is reflected.

Event C: Pulse arrives at detector.

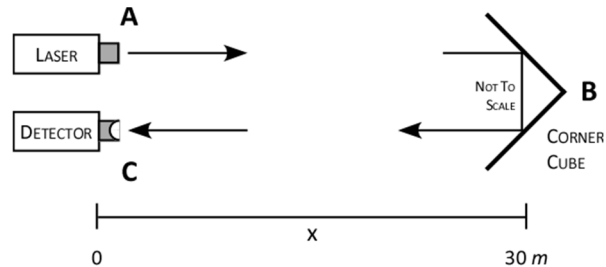


Figure 4

II. Describing the Experiment in the Home Frame

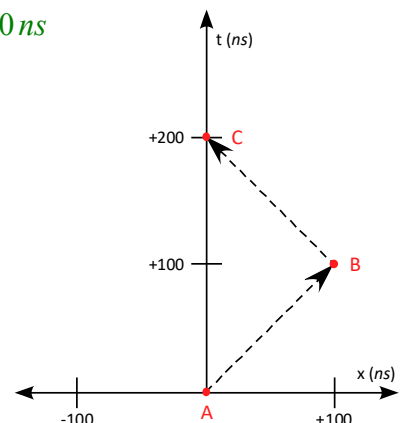
2. Assuming that the pulse travels at $v = 1$ (SR units), fill in the event table below:

| Event | x (ns) | t (ns) |
|-------|----------|----------|
| A | 0 | 0 |
| B | 100 | 100 |
| C | 0 | 200 |

Every 30 m translates to 100 ns:

$$30 \text{ m} \times \frac{1 \text{ s}}{3 \times 10^8 \text{ m}} = 1 \times 10^{-7} \text{ s} = 100 \text{ ns}$$

3. Now make a space-time diagram showing these events and the worldline of the light pulse. Use a scale of 1 block = 10 ns for both axes (keep them the same scale).

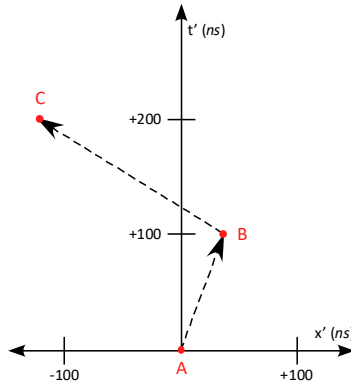


III. Describing the Experiment in the Other Frame – Newtonian World

4. Now consider another inertial frame moving in the +x direction with speed $\beta = 0.6$ and coincident with the

Home Frame at $t' = 0$. Fill in the event table using the Galilean Transformations: $\begin{cases} x' = x - \beta t \\ t' = t \end{cases}$

| Event | x' (ns) | t' (ns) |
|-------|-----------|-----------|
| A | 0 | 0 |
| B | 40 | 100 |
| C | -120 | 200 |



$$v_{out} = \frac{40 - 0}{100 - 0} = 0.40$$

$$v_{ret} = \frac{(-120) - 40}{200 - 100} = -1.6$$

$$v_{avg} = \frac{\text{Total Dist}}{\text{Total Time}} = \frac{40 + 160}{200} = 1$$

5. Make a new space-time diagram for the experiment, again keeping the axis scales the same.

6. Calculate the speed of the light pulse (in SR units) for both the outgoing and returning trip: $v = \frac{\Delta x'}{\Delta t'} = \frac{x'_{end} - x'_{start}}{t'_{end} - t'_{start}}$

7. Calculate the average speed of the light pulse in (SR units), noting that $average\ speed = \frac{\text{Total distance traveled (Event A to C)}}{\text{Total time}}$.

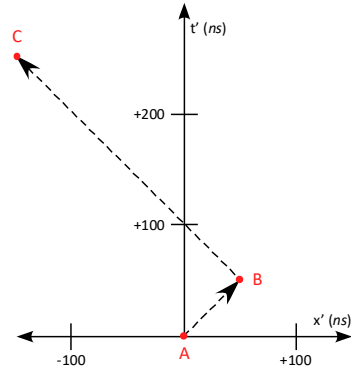
$$x' = \gamma(x - \beta t)$$

$$t' = \gamma(t - \beta x)$$

IV. Describing the Experiment in the Other Frame – Relativistic World

8. Recalculate the event table using the relativistic Lorentz Transformations, again with $\beta = 0.6$: $\gamma = \frac{1}{\sqrt{1 - \beta^2}}$

| Event | x' (ns) | t' (ns) |
|-------|-----------|-----------|
| A | 0 | 0 |
| B | 50 | 50 |
| C | -150 | 250 |



$$\gamma = \frac{1}{\sqrt{1 - (0.6)^2}} = 1.25$$

$$v_{out} = \frac{50 - 0}{50 - 0} = 1$$

$$v_{ret} = \frac{(-150) - 50}{250 - 50} = -1$$

$$v_{avg} = \frac{\text{Total Dist}}{\text{Total Time}} = \frac{50 + 200}{250} = 1$$

9. Make a new space-time diagram for the experiment, again keeping the axis scales the same.

10. Calculate the speed of the light pulse for both the outgoing and returning trip.

11. Calculate the average speed of the light pulse, again recalling the definition of average speed.

Discussion:

- Restate the value of 'c' you measured in the hallway. What are some sources of error?
- Your report should include the filled-in event tables, graphs of the three space-time diagrams, and the calculations for the speed of the light pulse. End with a brief summary of the differences in the Newtonian and relativistic descriptions of the experiment.

Summary:

- **Newtonian:** time is absolute; speed varies, can be greater or less than c
- **Relativistic:** time varies; speed is absolute – can only equal c!