THE SIMPLE PENDULUM

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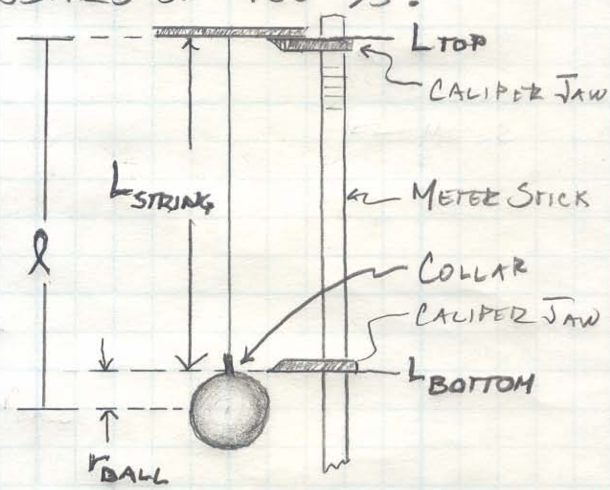
WE ARE GOING TO MEASURE THE PERIOD OF A SIMPLE PENDULUM TO SEE IF IT IS ACCURATELY DESCRIBED BY THE EQUATION FOR AN IDEAL PENDULUM:

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

T = PERIOD
ℓ = PENDULUM LENGTH (TO cm)
g = ACCELERATION OF GRAVITY

IF THIS FITS THE DATA WE WILL US IT TO DETERMINE g AND COMPARE IT TO THE STANDARD OF $980\ \mathrm{cm/s^2}$.



DATA

THE PENDULUM LENGTH IS

$$\ell = L_{STRING} + r_{BALL}$$

- WE MEASURED THE DIAMETER OF THE BALL WITH A CALIPER

$$d_{BALL} = 3.750\ \mathrm{cm}$$

$$r_{BALL} = \frac{d_{BALL}}{2} = 1.875\ \mathrm{cm}$$

- THE LENGTH OF THE STRING WAS VARIED (SEE TABLE)

- THE PERIOD OF THE PENDULUM AT EACH LENGTH WAS MEASURED WITH A PAIR OF PHOTOGATES SET ON "PEND." THE COLLAR JUST ABOVE THE BALL WAS USED TO BREAK THE PHOTOGATE BEAMS. MEASUREMENTS AT EACH LENGTH WERE TAKEN UNTIL THEY "SETTLED DOWN" THEN AVERAGED. THE DATA ARE LISTED ON THE NEXT PAGE.

PERIOD OF THE SIMPLE PENDULUM AT VARIOUS LENGTHS

L_{TOP} (cm)	L_{BOTTOM} (cm)	$L_{STRING} = L_{BOT} - L_{TOP}$ (cm)	T (SEC)	⟨T⟩ (cm)	$\ell = L_{STRING} + r_{BALL}$ (cm)
10.0	90.0	80.0	1.813 1.814 1.812 1.815 1.813 1.813 1.812	1.813	81.875 ≈ 81.9
	15.0	5.0	0.543 0.542 0.542 0.541 0.541	0.542	6.9
	70.0	60.0	1.575 1.574 1.575 1.575 1.574	1.575	61.9
	50.0	40.0	1.296 1.296 1.296 1.296 1.296	1.296	41.9
	30.0	20.0	0.937 0.936 0.935 0.937 0.936	0.936	21.9
	20.0	10.0	0.694 0.694 0.694 0.694 0.694 0.694	0.694	11.9

ANALYSIS

WE PLOTTED ⟨T⟩ AS A FUNCTION OF ℓ BY HAND (SEE MILLER) AND USING KALIEDA GRAPH (SEE ATTACHED). WITH KALEIDA-GRAPH, WE FIT A SQUARE ROOT FUNCTION TO THE DATA TO COMPARE IT TO THE IDEAL PENDULUM FOR WHICH

$$T = 2\pi\sqrt{\frac{\ell}{g}} = \frac{2\pi}{\sqrt{g}}\sqrt{\ell} = c\sqrt{\ell}.$$

KALEIDA GRAPH GAVE THE EQUATION OF THE CURVE AS

$$y = a x^{0.5}, \quad a = 0.20052$$

(SEE GRAPH).

THE COEFFICIENT FROM KALEIDAGRAPH SHOULD BE EQUAL TO "c" FOR THE IDEAL PENDULUM ($a = c$)

$$c = \frac{2\pi}{\sqrt{g}} \sim \frac{1}{\sqrt{\frac{m}{s^2}}} \sim \frac{s}{\sqrt{m}}$$

THUS

$$c = 0.20052 \ s/\sqrt{m}.$$

REARRANGING THIS TO GET g GIVES

$$g = \frac{4\pi^2}{c^2} = \frac{4\pi^2}{(0.20052)^2} = 981.8 \ \frac{cm}{s^2}$$

g MEASURED BY SIMPLE PENDULUM

COMPARING THIS TO THE STANDARD $g = 980 \ cm/s^2$ GIVES

$$\text{PERCENT DIFFERENCE} = \frac{(981.8 - 980)}{\frac{(981.8 + 980)}{2}} \times 100$$

$$\text{PERCENT DIFFERENCE} = 1.88\%$$

COMPARING g TO STANDARD

CONCLUSION

THE DATA SHOWED A SQUARE ROOT DEPENDENCE IN THE HAND GRAPH, INDICATING THAT THE EQUATION FOR THE IDEAL PENDULUM DESCRIBED OUR SIMPLE PENDULUM QUITE WELL.

KALEIDAGRAPH GAVE THE COEFFICIENT IN $T = c\sqrt{l}$

$$c = 0.20052 \ s/\sqrt{m}$$

YIELDING

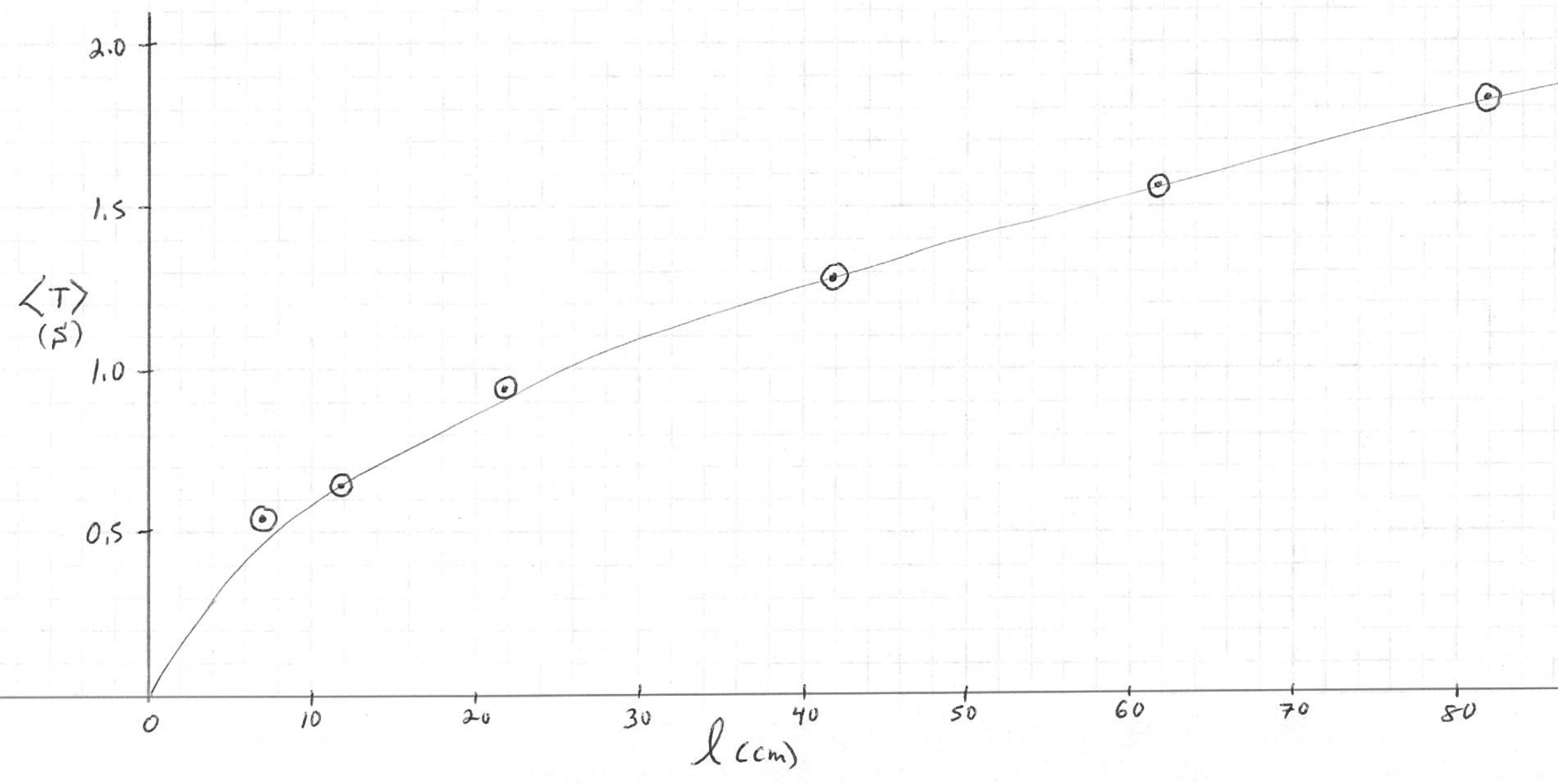
$$g = 981.8 \ cm/s^2$$

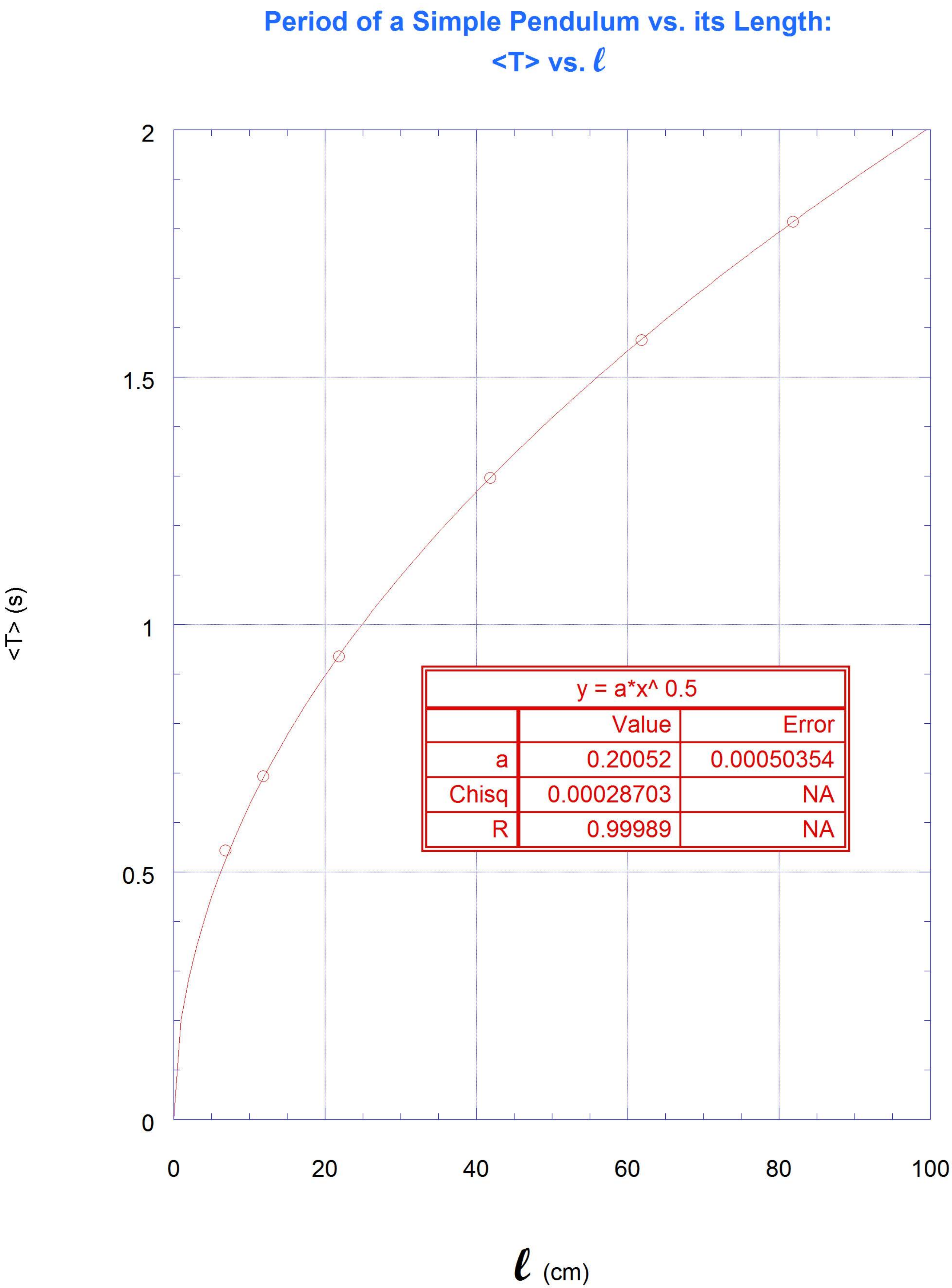
WHICH DIFFERS FROM STANDARD BY 1.88%. THIS SEEMS QUITE A GOOD MEASUREMENT.

TO IMPROVE THE ACCURACY, THE ADJUSTMENT AND MEASUREMENT OF THE PENDULUM LENGTH NEEDS TO BE IMPROVED WITH SOME SORT OF FINE INSTRUMENT — PERHAPS A DIGITAL SET-UP.

Period of Simple Pendulum vs. its Length

$\langle T \rangle$ vs. l




Period of a Simple Pendulum vs. its Length:
<T> vs. ℓ
2
1.5
1
0.5
0
0
20
40
60
80
100
<T> (s)
ℓ (cm)
y = a*x^ 0.5
Value
Error
a
0.20052
0.00050354
Chisq
0.00028703
NA
R
0.99989
NA