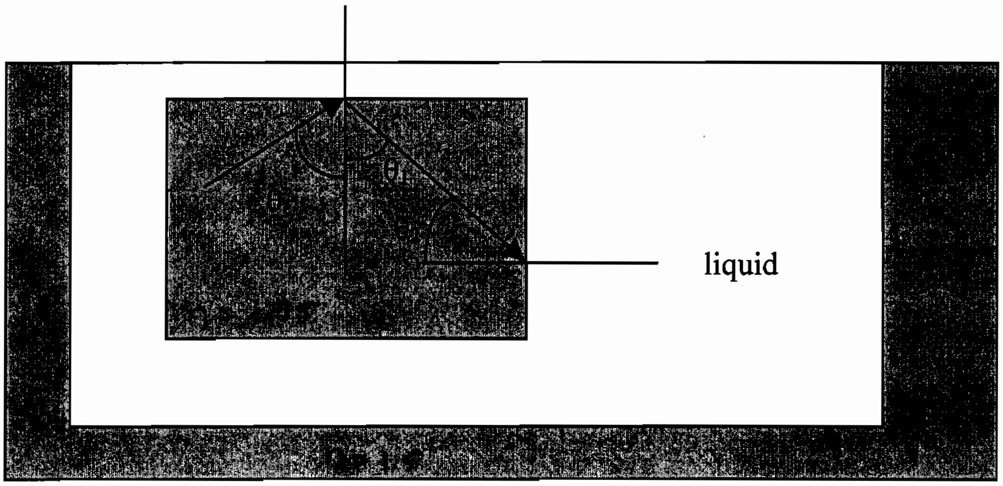
1.A sapphire gem (n=1.75) is submerged in an unknown liquid held in a glass beaker of index n=1.5. If a light ray coming from inside the gem is incident on the liquid interface at the critical angle of 57° as shown below

a. What is the index of refraction of the liquid?
b. Determine the angles θ1 and θ2 as drawn below.
c. Can the light exit the gem on the right side? If so, what is the angle the light makes in the liquid with respect to the normal?
d. What is Brewster's angle for a light ray starting inside the gem and incident on the liquid interface?



a) $\theta_c = 57°$ $\quad \sin\theta_c = \left(\frac{n_t}{n_i}\right) \rightarrow n_t = n_i \sin\theta_c = 1.75 \sin 57 = \boxed{1.468 = n_{liquid}}$

b) $\theta_i = \theta_R$ so $\theta_1 = 57°$ $\quad \theta_1 + \theta_2 = 90°$ so $90 - 57 = 33°$

c) $n_i \sin\theta_i = n_t \sin\theta_t$

$$\theta_t = \sin^{-1}\left(\frac{n_i}{n_t}\sin\theta_i\right) = \sin^{-1}\left(\frac{1.75}{1.468}\sin 33\right) = 40.4°$$

d) $\tan\theta_B = \left(\frac{n_t}{n_i}\right)$ $\quad \theta_B = \tan^{-1}\left(\frac{n_t}{n_i}\right) = \tan^{-1}\left(\frac{1.468}{1.75}\right) = 39.8°$

2. Chlorine 36 is unstable. Write down the decay equation for Chlorine 36 if it were to decay via the following processes:
   a. Alpha decay
   b. Beta minus decay
   c. Electron capture
   d. Beta plus decay
   e. Show whether or not Chlorine 36 can undergo beta plus decay. If it can what is the maximum kinetic energy available for the beta plus particle?

α: ${}^{36}_{17}Cl \rightarrow {}^{32}_{15}P + {}^{4}_{2}He$

β^-: ${}^{36}_{17}Cl \rightarrow {}^{36}_{18}Ar + {}^{0}_{-1}e + \bar{\nu}$

ec. ${}^{36}_{17}Cl + {}^{0}_{-1}e \rightarrow {}^{36}_{16}S + \nu$

β^+: ${}^{36}_{17}Cl \rightarrow {}^{36}_{16}S + {}^{0}_{+1}e + \nu$

$\Delta m = m_{\text{sulfur atom}}$ 35.967081u
$+ 2m_e$ + 2x0.00054 86u
$- m_{\text{Cl atom}}$ − 35.968307u

-1.288×10^{-4} u

since this is negative it can undergo β^+ decay

$KE_{max} = \Delta mc^2$

$= 1.288 \times 10^{-4}\,u \times \frac{931.5\,MeV}{c^2 u} c^2$

$= 0.12\,MeV$

6. Light incident on a pair of slits produces an interference pattern on a screen 2.50m from the slits. If the slit separation is 0.0150cm and the distance between adjacent bright fringes in the pattern is 0.760cm, what is the wavelength of the light?

$\theta = \tan^{-1}(Y/D) = \tan^{-1}\left(\frac{0.760\,cm}{250\,cm}\right) = 0.174$

$m\lambda = d\sin\theta$

$\lambda = 0.015 \times 10^{-2}\,m \; \sin(0.174)$

$= 4.55 \times 10^{-7}\,m$

$\boxed{\lambda = 455\,nm}$

3. Naturally-occurring carbon in the atmosphere contains a small amount of ^{14}C, which is radioactive. (You will need Appendix B for this problem.)

a) What kind of decay does ^{14}C undergo, and what does it decay to? β^-
b) What is the decay constant for ^{14}C?
c) There are 6.5×10^{10} ^{14}C atoms in a gram of carbon. What will the activity per gram of carbon be in a living sample (one in which respiration is continually bringing atmospheric carbon into the tissues)?
d) If the organism stops respiring (dies!) what will be activity per gram of carbon be after 8600 years?

a) $^{14}_{6}C \longrightarrow {}^{0}_{-1}e + {}^{14}_{7}N + \bar{\nu}$

b) $T_{1/2} = 5730y$ $\quad \tau = \frac{T_{1/2}}{\ln 2}$ $\quad \lambda = \frac{1}{\tau} = \frac{\ln 2}{T_{1/2}}$

need to convert $T_{1/2}$ to s

$$T_{1/2} = 5730y \times \frac{364d}{y} \times \frac{24h}{d} \times \frac{3600s}{h} = 1.8 \times 10^{11} s$$

$$\lambda = \frac{\ln 2}{1.8 \times 10^{11} s} = \boxed{3.8 \times 10^{-12}\ 1/s = \lambda}$$

c) $N = 6.5 \times 10^{10}$ $\quad$ Activity / gram $= \frac{R}{1g}$

$$\frac{R}{g} = \frac{N\lambda}{1g} = \frac{(6.5 \times 10^{10} \text{atoms})(3.8 \times 10^{-12}\ 1/s)}{1g}$$

$$= \frac{0.247\ 1/s}{g} = 0.247 \frac{Bq}{g}$$

$$\tau = \frac{5730y}{\ln 2} = 8266y$$

$$R = R_0 e^{-t\lambda} = R_0 e^{-t/\tau}$$

$$\frac{R}{g} = 0.247 \frac{Bq}{g} e^{-8266/8600} = 9.44 \times 10^{-2} \frac{Bq}{g}$$

4. You have a diverging lens with a focal length of magnitude 10 cm. Answer the questions below for an object that is placed 15 cm in front of the lens. Justify each of your answers!

a e. Calculate where is the image located?
b f. Is it enlarged or diminished? By how much?
c g. Is it real or virtual?
d h. Is it upright or upside down
e i. Sketch a ray diagram for this system.

$f = -10\,cm \qquad p = +15\,cm$

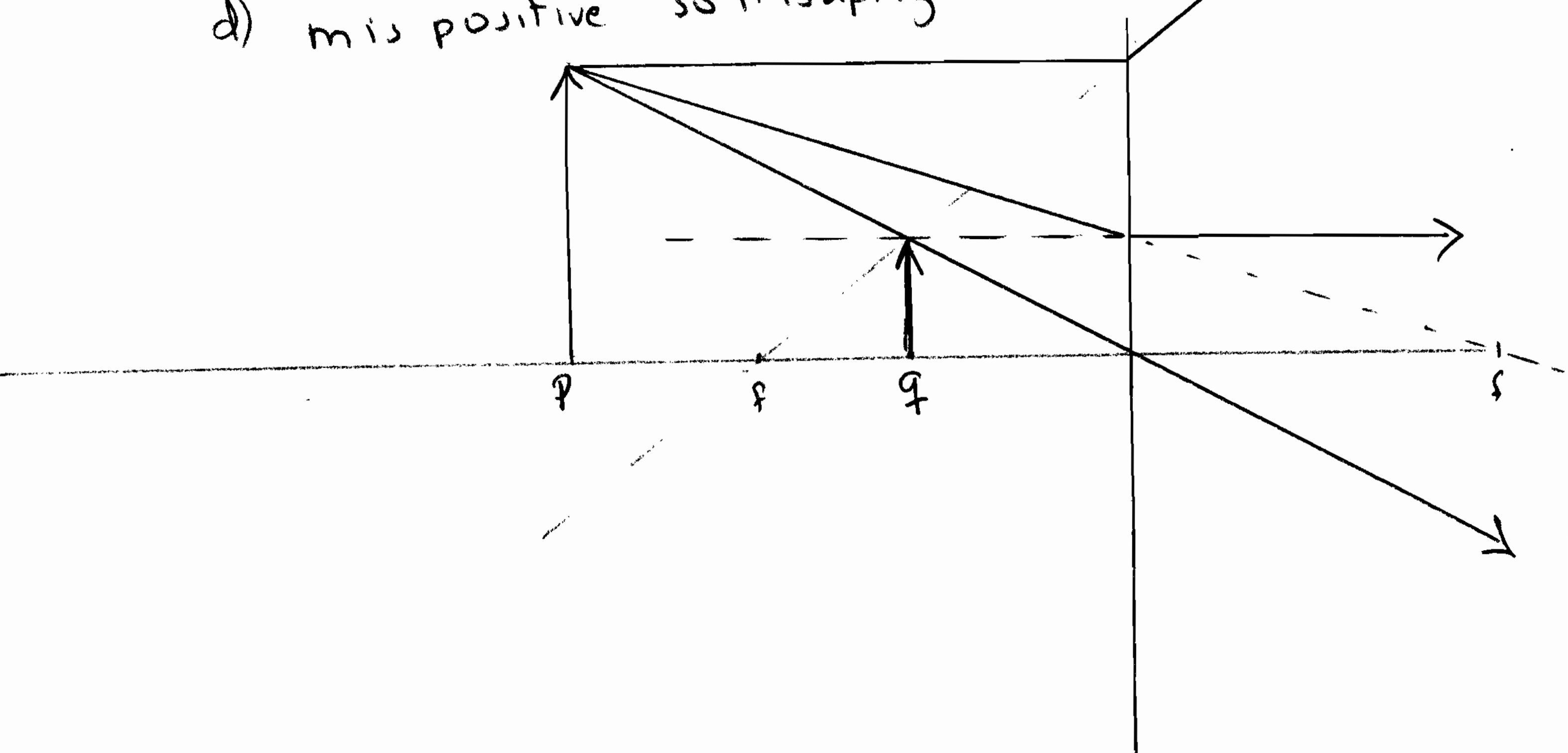
a) $\frac{1}{f} = \frac{1}{p} + \frac{1}{q} \qquad \frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{p-f}{fp}$

$q = \frac{fp}{p-f} = \frac{(-10cm)(15cm)}{15cm - (-10cm)} = \frac{-150cm^2}{25cm} = \boxed{-6cm = q}$

b) $m = -\frac{q}{p} = \frac{-(-6cm)}{15cm} = +0.4$ diminished by 0.4 times

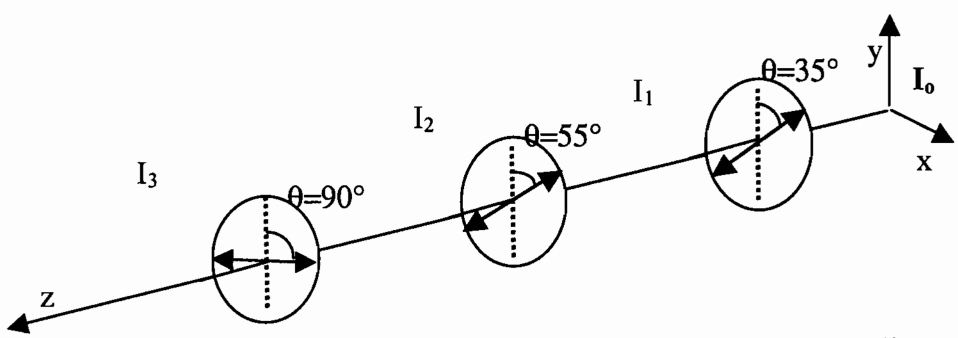
c) q is negative so it is virtual

d) m is positive so it is upright



5. Below light is shown traveling in the z direction.
   a. Describe the directions is it possible for this light to be polarized.
   b. Light is unpolarized with an intensity of I_o traveling in the z-direction. The light encounters a series of three polarizers with transmission axes as shown below, what fraction of the incident light gets through the polarizers?
   c. In what direction is the light in part (b) polarized when it exits the third polarizer?
   d. If you were to put in a fourth polarizer to reduce the intensity of the light to 0.1 I_o, what angle should it be placed **relative to the y-axis?**

a) I_o is traveling in the z-direction so it can be polarized in the x-y plane



b) $I_1 = \frac{1}{2} I_o$ because ½ of unpolarized Light goes through a polarizer

$I_2 = I_1 \cos^2\theta$ $\qquad \theta = 55° - 35° = 20°$

$= \frac{1}{2} I_o \cos^2 20 =$

$I_3 = I_2 \cos^2\theta$ $\qquad \theta = 90 - 55 = 35°$

$= 0.44 I_o \cos^2 35 =$ $\boxed{0.3 I_o = I_3}$

c) polarized in the x direction

$I_4 = I_3 \cos^2\theta$

$\cos^2\theta = \frac{I_4}{I_3}$

$\cos\theta = \sqrt{\frac{I_4}{I_3}}$

$\theta = \cos^{-1}\sqrt{\frac{I_4}{I_3}}$

$= \cos^{-1}\sqrt{\frac{0.1}{0.3}}$

$= 54.7°$ relative to the x-axis

$I_4 = 0.1 I_o$

$I_3 = 0.3 I_o$