57. (a) For a converging lens with a focal length of 3.50 cm, find the object distance that will result in an inverted image with an image distance of 5.00 cm. Use a ray diagram to verify your calculations. (b) Is the image real or virtual? (c) What is the magnification?

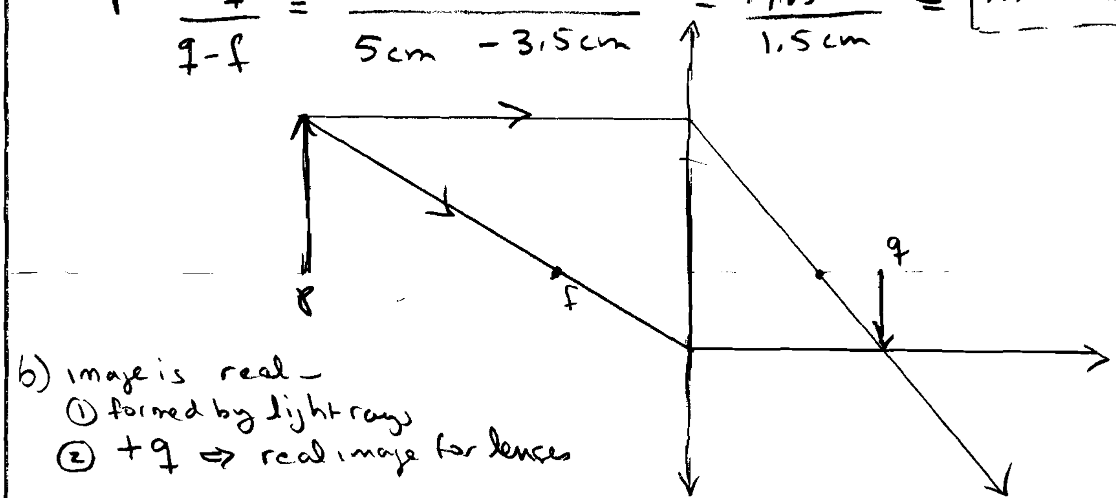
$f = +3.50$ cm

$q = 5.00$ cm

inverted image means m is negative
$m = -q/p$ so q and p have the same sign.

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q} \Rightarrow \frac{1}{p} = \frac{1}{f} - \frac{1}{q} = \frac{q}{fq} - \frac{f}{fq} = \frac{q-f}{fq}$$

$$p = \frac{fq}{q-f} = \frac{(3.5\text{cm})(5\text{cm})}{5\text{cm} - 3.5\text{cm}} = \frac{17.5\text{cm}^2}{1.5\text{cm}} = \boxed{11.7\text{cm} = p}$$



b) image is real –
① formed by light rays
② $+q \Rightarrow$ real image for lenses

c) $m = -q/p = -1/2$

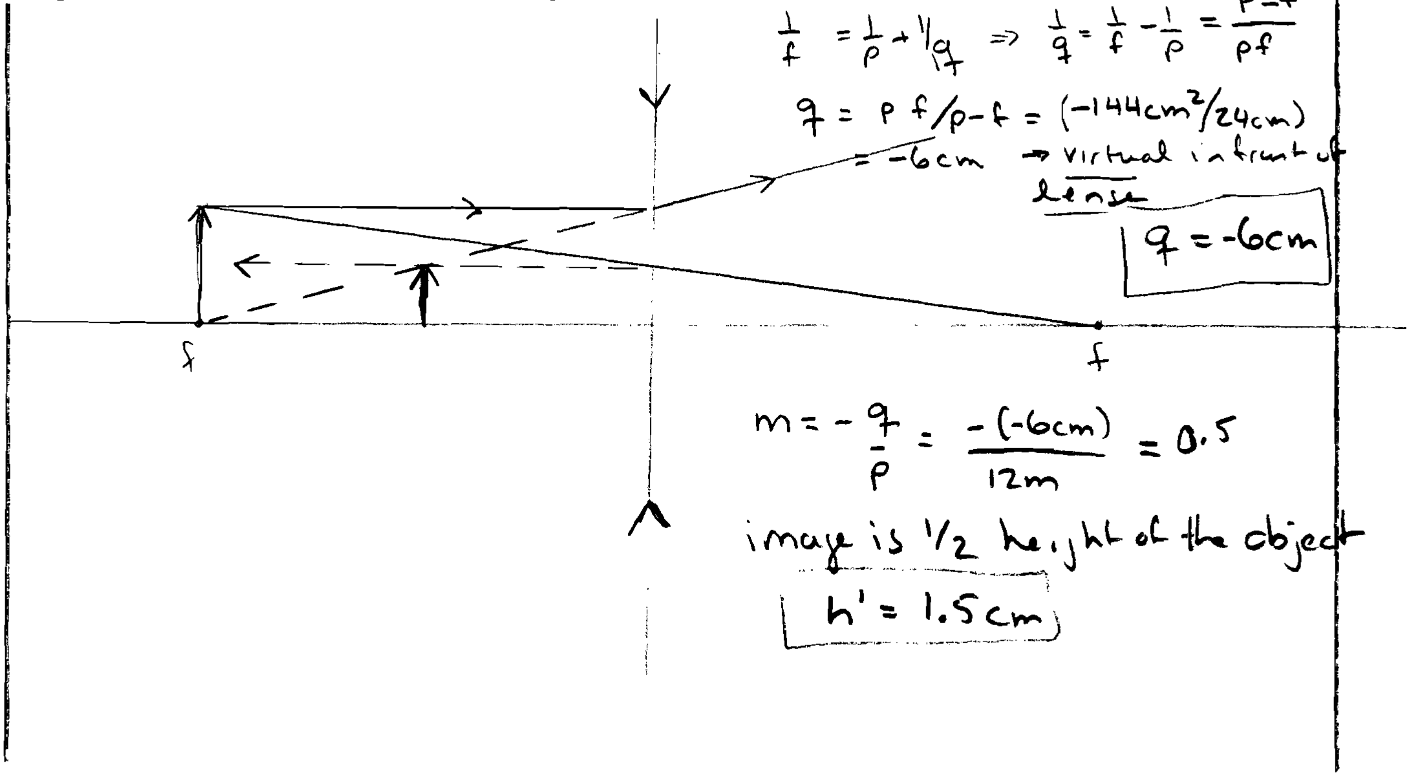
63. An object of height 3.00 cm is placed 12.0 cm from a diverging lens of focal length −12.0 cm. Draw a ray diagram to find the height and position of the image.

$f = -12.0$ cm $h = 3.00$ cm
$p = 12.0$ cm $q = ?$

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q} \Rightarrow \frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{p-f}{pf}$$

$q = pf/(p-f) = (-144\text{cm}^2/24\text{cm})$
$= -6$cm → virtual in front of lense

$\boxed{q = -6\text{cm}}$



$$m = -\frac{q}{p} = \frac{-(-6\text{cm})}{12\text{cm}} = 0.5$$

image is 1/2 height of the object

$\boxed{h' = 1.5\text{cm}}$

65. A converging lens has a focal length of 8.00 cm. (a) What are the image distances for objects placed at these distances from the thin lens: 5.00 cm, 14.0 cm, 16.0 cm, 20.0 cm? In each case, describe the image as real or virtual, upright or inverted, and enlarged or diminished in size. (b) If the object is 4.00 cm high, what is the height of the image for the object distances of 5.00 cm and 20.0 cm?

$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$

$\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{p-f}{fp}$

$q = \frac{fp}{p-f}$ $m = -\frac{q}{p}$

q − virtual, m < 1 smaller, m − upside down
q + real, m > 1 bigger, m + rightside up

$p = 5.00$ cm $f = 8.00$ cm

$q = \frac{fp}{p-f}$ $q = \frac{(8)(5)}{5-8} = \frac{40 \text{ cm}^2}{-3 \text{ cm}}$

$q = -13.3$ cm virtual

$m = -q/p = -\frac{(-13.3)}{5} = 2.67$ bigger and upright

$p = 14.0$ cm

$q = \frac{fp}{p-f} = \frac{(8)(14)}{14-8} = +18.7$ cm real $m = -\frac{q}{p} = -\frac{18.7}{14} = -1.33$ bigger upside down

$p = 16.0$ cm

$q = \frac{fp}{p-f} = \frac{(8)(16)}{16-8} = \frac{128}{8} = 16$ cm $m = -\frac{q}{p} = -\frac{16}{16} = -1$ upside down same size

$p = 20$ cm

$q = \frac{fp}{p-f} = \frac{(8)(20)}{20-8} = \frac{160 \text{ cm}^2}{12 \text{ cm}} = 13.3$ cm $m = \frac{-q}{p} = -\frac{13.3}{20} = -0.66$ upside down smaller

b) $m = h'/h$ $h = 4.00$ cm

$p = 5$ cm

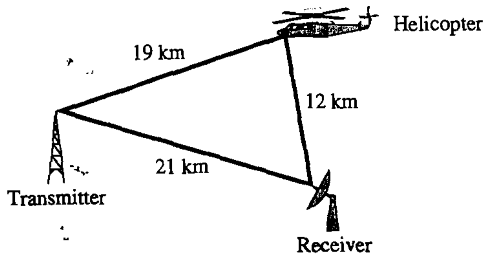
$h' = mh$
$= (2.67)(4.0 \text{ cm})$
$= 10.7$ cm

$p = 20$ cm

$h' = (0.66)(4.00 \text{ cm})$
$= -2.67$ cm

p (cm)	q (cm)	Virtual / real	upside down / rightside up	large/smaller/same
5	-13.3	virtual	upright	bigger 2.67 = m
14	18.7	real	upside down	bigger m = -1.33
16	16	real	upside down	same size m = -1
20	13.3	real	upside down	smaller m = -0.667

1. A 60-kHz radio transmitter sends an electromagnetic wave to a receiver 21 km away. The signal also travels to the receiver by another path where it reflects from a helicopter as shown. Assume that there is a 180° phase shift when the wave is reflected. (a) What is the wavelength of this EM wave? (b) Will this situation give constructive interference, destructive interference, or something in between?



Interference is caused by the path difference

Path 1 $= d_1$ = Transmitter to receiver

$d_1 = 21$ km

Path 2 $= d_2$ = Transmitter → helicopter → receiver

$d_2 = 19\text{ km} + 12\text{ km}$
$= 31$ km

Path difference $= \Delta d = d_2 - d_1 = 31\text{ km} - 21\text{ km} = 10\text{ km}$

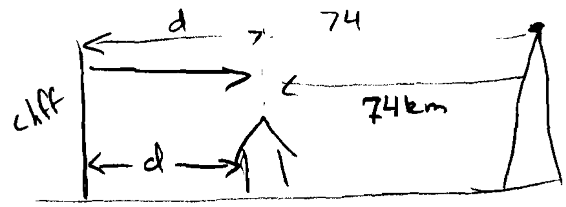
What is the phase difference? $\Delta\phi = \Delta d + \frac{\lambda}{2}$

(a) $\lambda = \frac{c}{f} = \frac{3\times10^8 \text{ m/s}}{60{,}000 \text{ Hz}} = 5000 \text{ m} = 5 \text{ km}$

So $\Delta\phi = 10\text{ km} + 2.5\text{ km} = 12.5\text{ km} \Rightarrow \frac{\Delta\phi}{\lambda} = 2.5$ = half-integer

∴ destructive interference

2. A steep cliff west of Lydia's home reflects a 1020-kHz radio signal from a station that is 74 km due east of her home. If there is destructive interference, what is the minimum distance of the cliff from her home? Assume there is a 180° phase shift when the wave reflects from the cliff.



$\lambda = c/f = 3\times10^8 \text{ m/s} / 1{,}020{,}000 \text{ Hz} = 294 \text{ m}$

Path difference $= d_2 - d_1 = (74\text{ km} + 2d) - 74\text{ km} = 2d$

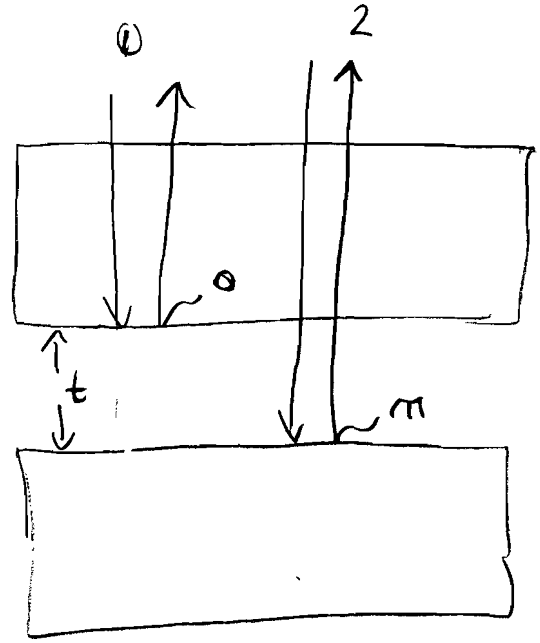
since there is a phase shift of 180° ($\lambda/2$) at the cliff

we need $2d = n\lambda$ where n is an integer

smallest $n = 1$

$d = \lambda/2 = 294\text{ m}/2 = \boxed{147\text{ m} = d}$

14. At a science museum, Marlow looks down into a display case and sees two pieces of very flat glass lying on top of each other with light and dark regions on the glass. The exhibit states that monochromatic light with a wavelength of 550 nm is incident on the glass plates and that the plates are sitting in air. The glass has an index of refraction of 1.51. (a) What is the minimum distance between the two glass plates for one of the dark regions? (b) What is the minimum distance between the two glass plates for one of the light regions? (c) What is the next largest distance between the plates for a dark region? [*Hint:* Do not worry about the thickness of the glass plates; the *thin* film is the air between the plates.]



Rays ① and ② can interfere

Path difference $\Delta d = d_2 - d_1 = 2t$
where t is the gap.

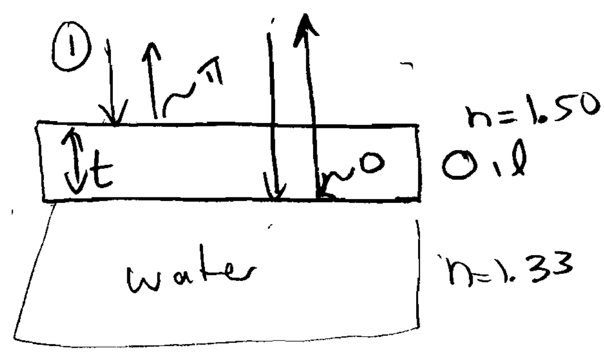
Ray 2 also has a $\lambda/2$ phase shift when it reflects off the air glass interface

a) Destructive interference occurs when the $\Delta d + \lambda/2 = (m + 1/2)\lambda$ m integer
minimum is when $m = 0 \Rightarrow \Delta d + \lambda/2 = \lambda/2$
so minimum thickness for destructive interference $= 0$

b) Constructive interference
$\Delta d + \lambda/2 = m\lambda$ minimum $\Rightarrow m = 1$
$2t + \lambda/2 = \lambda$ $2t = \lambda/2$ $t = \lambda/4 = \frac{550}{4} = 137\,nm$

c) Next darkest
$\Delta d + \lambda/2 = (m + 1/2)\lambda$ where $m = 1$
$2t = \lambda$ $t = \lambda/2 = \frac{550\,nm}{2} = 275\,nm$

17. A thin film of oil ($n = 1.50$) of thickness 0.40 μm is spread over a puddle of water ($n = 1.33$). For which wavelength *in the visible spectrum* do you expect constructive interference for reflection at normal incidence?



Constructive interference

$\Delta d + \lambda/2 = n\lambda$ $\Delta d = 2t$
$\Delta d = (m - 1/2)\lambda$

$\lambda = \frac{2t}{(m - 1/2)}$. But this is λ in the film $\lambda_{film} = \lambda_0/n$

$\lambda_0 = \frac{2tn}{m - 1/2} = \frac{2(0.4\times10^{-6}\,m)(1.5)}{1/2} = 2400\,nm$ (m=1)

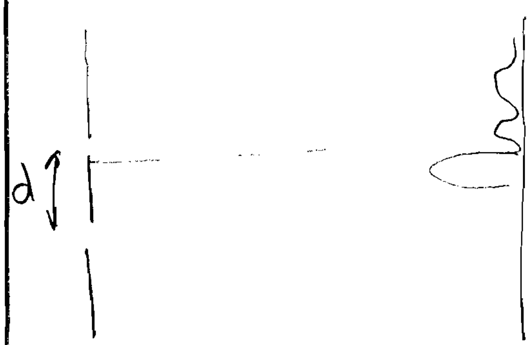
m=2: 800 nm; m=3: 480 nm (visible); m=4: 343 nm

26. Light of 650 nm is incident on two slits. A maximum is seen at an angle of 4.10° and a minimum of 4.78°. What is the order *m* of the maximum and what is the distance *d* between the slits?

$\lambda = 650\,nm$

$\theta = 4.10°$ maximum

$\theta = 4.78°$ minimum



maxima $d\sin\theta = m\lambda$

minima $d\sin\theta = (m+1/2)\lambda$

Divide them

$$\frac{d\sin\theta_{min}}{d\sin\theta_{max}} = \frac{(m+1/2)\lambda}{m\lambda}$$

$$\frac{\sin\theta_{min}}{\sin\theta_{max}} = 1 + 1/2m \Rightarrow \left(\frac{1}{2m}\right) = \left(\frac{\sin\theta_{min}}{\sin\theta_{max}} - 1\right)$$

$$\left(\frac{1}{2m}\right)^{-1} = \left(\frac{\sin\theta_{min} - \sin\theta_{max}}{\sin\theta_{max}}\right)^{-1}$$

$$2m = \frac{\sin\theta_{max}}{\sin\theta_{min} - \sin\theta_{max}}$$

$$m = \frac{1/2 \sin 4.10}{\sin 4.78 - \sin 4.10} =$$

$m = 3$

$$d = m\lambda/\sin\theta$$

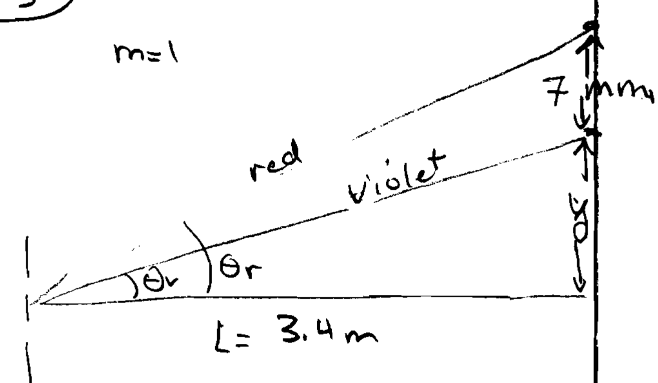
$$= \frac{3(650\times10^{-9}m)}{\sin 4.10}$$

$$= 2.73\times10^{-5}m$$

$d = 27.3\,\mu m$

27. You are given a slide with two slits cut into it and asked how far apart the slits are. You shine white light on the slide and notice the first-order color spectrum that is created on a screen 3.40 m away. On the screen, the red light with a wavelength of 700 nm is separated from the violet light with a wavelength of 400 nm by 7.00 mm. What is the separation of the two slits?

$m=1$



$d\sin\theta_r = \lambda_{red}$ $\quad \tan\theta_r = \frac{y+7mm}{L}$

$d\sin\theta_v = \lambda_{violet}$ $\quad \tan\theta_v = \frac{y}{L}$

small angle approximation ~ $\sin\theta = \tan\theta$

$d\left(\frac{y+7mm}{L}\right) = \lambda_{red}$

$d\,y/L = \lambda_v$

$$\lambda_{red} - \lambda_v = d\frac{(7mm)}{L}$$

$$d = \frac{(\lambda_{red} - \lambda_v)L}{d} = \frac{(700\times10^{-9} - 400\times10^{-9})(3.4)}{7\times10^{-3}m}$$

$= 1.46\times10^{-4}m$ or $0.146\,mm$

36. A grating has exactly 8000 slits uniformly spaced over 2.54 cm and is illuminated by light from a mercury vapor discharge lamp. What is the expected angle for the third-order maximum of the green line ($\lambda = 546$ nm)?

Grating equation
$d \sin\theta = m\lambda$

$d =$ cm/slit

8000 slits over 2.54 cm

$d = \frac{2.54\text{cm}}{8000} = 3.18 \times 10^{-4}$ cm $= 3.18 \times 10^{-6}$ m

$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{546\times10^{-9}\text{ m } 3}{3.18\times10^{-6}\text{ m}}\right) = 31°$

(m = 3)

37. A red line (wavelength 630 nm) in the third order overlaps with a blue line in the fourth order for a particular grating. What is the wavelength of the blue line?

d is unchanged.
Since they overlap,
θ is the same

$d \sin\theta = m_{red}\lambda_{red} = m_{blue}\lambda_{blue}$

$\lambda_{blue} = \frac{m_{red}}{m_{blue}} \lambda_{red}$

$\lambda = 630\text{nm}\left(\frac{3}{4}\right) =$ **470 nm $= \lambda_{blue}$**

38. Red light of 650 nm can be seen in three orders in a particular grating. About how many slits per centimeter does this grating have?

$m\lambda = d\sin\theta$ biggest order when $\theta \sim 90$

$d = \frac{m\lambda}{\sin\theta} = \frac{3(650\text{nm})}{\sin 90}$

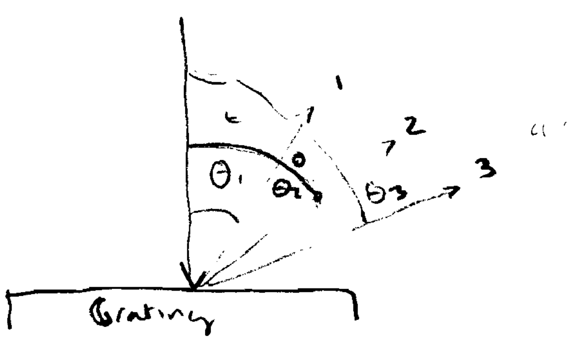
$d \cong 1.95 \times 10^{-6}$ m/slit

$1/d = 512800$ slits/m

$1/d \cong 5128$ slits/cm

~ 5000 slits/cm

$\lambda = 650$nm
$m = 3$
We don't know θ



θ increases with order m

biggest possible $\theta = 90°$
This corresponds to smallest d