Chapter 29 P: 5, 6, 10, 13, 22, 23, 25, 26, 27, 33, 41, 68, 69

Perodic table on the back cover

5. Write the symbol (in the form ${}^{A}_{Z}X$) for the isotope of potassium with 21 neutrons.

Potassium = K 19 = Z from perodic table

N = 21

$A = Z + N = 19 + 21 = 40$ $\boxed{{}^{40}_{19}K}$

6. How many neutrons are found in a ${}^{35}Cl$ nucleus?

Cl $Z = 17$ $A = N + Z = 35$ $N = 35 - 17 = \boxed{18}$

~~7.~~ How many protons are found in a ${}^{136}Xe$ nucleus?

NOT ASSIGNED

$\boxed{Xe\ Z = 54}$ looked up in periodic table

10. What is the binding energy of an α particle (a ${}^{4}He$ nucleus)? The mass of an α particle is 4.001 51 u.

$Z = 2$ $N = 2$

In general

$E_B = \Delta m c^2 = (Z m_p + N m_n - m_{nucleus})c^2$ $m_{nucleus} = m_{atom} - Z m_e$

$= [Z m_p + N m_N - \underline{(m_{atom} - Z m_e)}]c^2 \times 931 \frac{MeV}{c^2 u}$

But here we are given the nuclear mass m_α so we do not need to subtract electron masses

$= [2(1.0072765 u) + 2(1.0086649 u) - 4.001514 u]$

$= [0.03037 u] \times 931.5 MeV/(uc^2)$

$E_B = 28.29$ MeV

13. (a) Find the binding energy of the ${}^{16}O$ nucleus. (b) What is the average binding energy per nucleon? Check your answer using Fig. 29.2.

O $Z = 8$ $N = A - Z = 8$

From Appendix B

$m_{atom} = 15.9949146 u$

a) $E_B = \Delta m c^2 = (Z m_p + N m_n - m_{nucleus})c^2$

$= [Z m_p + N m_N - (m_{atom} - Z m_e) c^2] \times 931.5 MeV/c^2 u$

$= [8(1.0072765 u) + 8(1.0086649 u) - 15.9949146 u + 8(0.000548580 u)] 931.5 \frac{MeV}{c^2 u}$

$= [0.1370 u] \times 931.5 MeV/c^2 u$

$\boxed{E_B = 127.6 MeV}$

b) $E_B/A = 127.6 MeV/16 = 7.98$ MeV/nucleon

c) matches graph

22. Thorium-232 ($^{232}_{90}$Th) decays via α decay. Write out the reaction and identify the daughter nuclide.

$$^{A}_{Z}P \rightarrow {}^{A-4}_{Z-2}D + {}^{4}_{2}\alpha$$

A = 232   A − 4 = 228

Z = 90   Z − 2 = 88 → Ra $^{228}_{88}Ra$

$$\boxed{^{232}_{90}Th \rightarrow {}^{228}_{88}Ra + {}^{4}_{2}\alpha}$$

23. Write out the reaction and identify the daughter nuclide when $^{22}_{11}$Na decays by electron capture.

$$^{A}_{Z}P + e^- \rightarrow {}^{A}_{Z-1}D + \nu$$

$^{22}_{11}Na$ A = 22   Z = 11   Z − 1 = 10 → Ne

$$^{22}_{11}Na + e^- \rightarrow {}^{22}_{10}Ne + \nu$$

25. Radium-226 decays as $^{226}_{88}\text{Ra} \rightarrow {}^{222}_{86}\text{Rn} + {}^{4}_{2}\text{He}$. If the $^{226}_{88}$Ra nucleus is at rest before the decay and the $^{222}_{86}$Rn nucleus is in its ground state, *estimate* the kinetic energy of the α particle. (Assume that the $^{222}_{86}$Rn nucleus takes away an insignificant fraction of the kinetic energy.)

Here we assume that the energy in the decay all goes into KE of the α.

From Appendix B

$$\Delta m = m\ {}^{222}_{86}Rn + {}^{4}_{2}He - {}^{226}_{88}Ra$$

$\Delta m = m_{FINAL} - m_{INITIAL}$

"−" Δm means the mass of the products is less than the mass of the parent and energy is released

"+" Δm means the decay is not spontaneous and Δmc² energy must be added for the decay to happen.

^{222}RN 222.0175705 u

$^{4}_{2}He$ + 4.0026032 u

− 226.0254026 u

− 5.2289 × 10⁻³ u

$$E = \Delta m c^2$$

$$= (5.2289\times10^{-3}\,u)c^2\ \frac{931.5\,MeV}{c^2 u}$$

$$\boxed{E = 4.871\ MeV}$$

26. Which decay mode would you expect for radioactive $^{31}_{14}$Si: α, β^-, or β^+? Explain. [*Hint:* Look at the neutron-to-proton ratio.]

α $^{31}_{14}Si \rightarrow ^{27}_{12}Mg$

β⁻ $^{31}_{14}Si \longrightarrow ^{31}_{13}Al + e^-$

β⁺ $^{31}_{14}Si \rightarrow ^{31}_{15}P + e^+$

$^{31}_{14}Si$ A = 31 Z = 14 N = A − Z = 17

small nuclides prefer Z≈N so we have too many neutrons

β^- converts a $p \rightarrow n$ wrong direction!

β^+ converts a $n \rightarrow p$ better

		α	β⁻	β⁺
	$^{31}_{14}Si$	$^{27}_{12}Mg$	$^{31}_{13}Al$	$^{31}_{15}P$
N/Z	$\frac{17}{14}$	$\frac{15}{12}$	$\frac{18}{13}$	$\boxed{\frac{16}{15}}$
	1.2	1.25	1.4	$\boxed{1.07}$

β^+

27. Calculate the maximum kinetic energy of the β particle when $^{40}_{19}$K decays via β^- decay.

max KE of β occurs when all of the energy released in the decay goes into KE of the β

$^{A}_{Z}P \rightarrow ^{A}_{Z+1}D + e^- + \bar{\nu}$

$^{40}_{19}K \rightarrow ^{40}_{20}Ca + e^- + \bar{\nu}$

$$E_B = \Delta m c^2 = (m_F - m_i)c^2$$

$$= [m_{D\,nucleus} + m_e - m_{P\,nucleus}]c^2$$

$$= [m_{D\,atom} - (Z+1)m_e + m_e - (m_{P\,ATOM} - Zm_e)]c^2$$

$$= [m_{D\,Atom} - m_{P\,atom}]c^2$$

$$= -0.0014075u\,c^2$$

$$= -0.0014075u\,c^2 \,\frac{931.5\,\text{MeV}}{c^2 u}$$

From Appendix B

$^{40}_{19}K$ 39.963 998 7 u

$^{40}_{20}Ca$ 39.962 591 2

$= -1.31$ MeV

↑

negative means that this is the amount of energy released in the decay

positive would mean that the system was bound and could not spontaneously decay without adding the amount of energy.

33. A certain radioactive nuclide has a half-life of 200.0 s. A sample containing just this one radioactive nuclide has an initial activity of 80,000.0 s^{-1}. (a) What is the activity 600.0 s later? (b) How many nuclei were there initially? (c) What is the probability per second that any one of the nuclei decays?

$R_0 = 80{,}000\ 1/s$

$T_{1/2} = 200\ s$

a) $R = ?$ b) $N_0 = ?$ c) $\lambda = ?$

a) $R = R_0 e^{-t/\tau}$ $\quad \tau = T_{1/2}/\ln 2 = \frac{200 s}{0.693} = 288.6 s$

$= (80{,}000\ 1/s)(e^{-600/288.6})$

$= (80{,}000\ 1/s)(0.125)$

$\boxed{R = 1\times10^{4}\ 1/s}$

OR

$R = R_0\left(\frac{1}{2}\right)^{t/T_{1/2}}$

$= 80{,}000(1/2)^{600/200}\ (1/s)$

$= 80000(1/2)^3\ 1/s$

$= 80{,}000\ 1/8 = 10{,}000\ 1/s$

b) $R = \lambda N$ so

$R_0 = \lambda N_0$ $\qquad \lambda = 1/\tau = \frac{\ln 2}{T_{1/2}} = 3.465\times10^{-3}\ \frac{decay}{s}$

$N_0 = R_0/\lambda$

$= 80{,}000/3.465\times10^{-3}\ \frac{decay}{s} = 2.3\times10^{7}$ Nuclei

c) Probability $= \lambda = 1/\tau = \frac{\ln 2}{T_{1/2}} = 3.465\times10^{-3}$ decays/s

✦41. In this problem, you will verify the statement (in Section 29.4) that the ^{14}C activity in a living sample is 0.25 Bq per gram of carbon. (a) What is the decay constant λ for ^{14}C? (b) How many ^{14}C atoms are in 1.00 g of carbon? One mole of carbon atoms has a mass of 12.011 g, and the relative abundance of ^{14}C is 1.3×10^{-12}. (c) Using your results from parts (a) and (b), calculate the ^{14}C activity per gram of carbon in a living sample.

a) $T_{1/2} = 5730$ years $\quad \lambda = \frac{\ln 2}{T_{1/2}} = 1.$

convert to seconds

$T_{1/2} = 5730\,y \times \frac{364 d}{y} \times \frac{24 h}{d} \times \frac{60 min}{h} \times \frac{60 s}{min} = 1.802\times10^{11} s$

$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{1.802\times10^{11} s} = 3.83\times10^{-12}\ 1/s$

b) $= N = \frac{mass}{mass/mole} \times N_A \times$ Relative abundance

C^{14} vs othe carbon isotopes

$= \frac{1.00 g}{12.011\ g/mol} \times 6.022\times10^{23}\ \frac{atoms}{mole} \times 1.3\times10^{-12}$

$= 6.5\times10^{10}$ atoms

c) $\frac{R}{mass} = \frac{R}{1.00 g} = \frac{\lambda N}{1.00 g} = \frac{3.83\times10^{-12}\ 1/s \cdot 6.5\times10^{10}\ atoms}{1.00 g} = 0.25\ \frac{Bq}{g}$

68. In 1988 the shroud of Turin, a piece of cloth that some people believe is the burial cloth of Jesus, was dated using ^{14}C. The measured ^{14}C activity of the cloth was about 0.23 Bq/g. According to this activity, when was the cloth in the shroud made?

$T_{1/2} = 5730y$ $\tau = \frac{T_{1/2}}{\ln 2} = 8267y$

$R_o = 0.25\ Bq/g$ $R = 0.23\ Bq/g$

$R = R_o e^{-t/\tau}$ $R/R_o = e^{-t/\tau}$ $\ln(R/R_o) = -t/\tau$

$t = -\tau \ln(R/R_o) = -(8267y)\ln\left(\frac{0.23}{0.25}\right) = 689$ years old

$1988 - 689 = 1299$ AD A TAD TOO Late

*69. Radon gas (Rn) is produced by the α decay of radium $^{226}_{88}Ra$ (a) How many neutrons and how many protons are present in the nucleus of the isotope of Rn produced by this decay? (b) In the air in an average size room in a student basement apartment in Ithaca, NY, there are about 10^7 Rn nuclei. The Rn nucleus itself is radioactive; it too decays by emitting an α particle. The half-life of Rn is 3.8 days. How many α particles per second are emitted by decaying Rn nuclei in the room?

$^{Z}_{A}P \rightarrow {}^{A-4}_{Z-2}D + \alpha$

For Daughter

$A = 226 - 4 = 222$

$Z = 88 - 2 = \boxed{86}$ protons

$N = A - Z = 222 - 86 = \boxed{136}$ neutrons

a) $^{226}_{88}Ra \rightarrow {}^{222}_{86}Rn + \alpha$

b) $N = 10^7$ $T_{1/2} = 3.8\ d$

$R = N\lambda = N/\tau = \frac{N \ln 2}{T_{1/2}} = \frac{10^7 \ln 2}{3.8d(24h/d)(3600s/h)} \cong 21 \frac{decays}{s}$

$21\ \alpha \frac{particles}{second}$ are emitted