We can treat circuits with capacitors in similar way to the circuits with resistors. We still find that voltages across parallel elements are the same. Now instead of having currents in series elements be the same, charges are the same. We can use a similar methodology where we reduce the sets of capacitors to an equivalent capacitance and use the ideas of series charge being the same and parallel voltages being the same. We can use the following set of equations.

In this circuit, $C_{1}=2.0 \mu \mathrm{~F}, C_{2}=1.0 \mu \mathrm{~F}, C_{3}=3.0 \mu \mathrm{~F}$, and $\mathrm{V}=20 \mathrm{~V}$.

$$
\begin{array}{cl}
Q=C V & \frac{1}{C_{\text {series }}}=\frac{1}{C_{1}}+\frac{1}{C_{2}}+\frac{1}{C_{3}} \\
& C_{| |}=C_{1}+C_{2}+C_{3}
\end{array}
$$



Begin by reducing $C_{2}$ and $C_{3}$ to an equivalent capacitance and redraw the circuit. Resolve the parallel set: $C_{23}=C_{2}+C_{3}$

$$
C_{23}=1.0+3.0=4.0 \mu \mathrm{~F}
$$



Next reduce the remaining capacitors to one and redraw the circuit.
Remaining capacitors are in series:

$$
\frac{1}{C_{123}}=\frac{1}{C_{1}}+\frac{1}{C_{23}}
$$

$$
C_{123}=\frac{C_{1} C_{23}}{C_{1}+C_{23}}=\frac{(2)(4)}{(2)+(4)}=1.33 \mu \mathrm{~F}
$$



Now use the idea that series capacitors have the same charge and parallel capacitors have the same voltage to find the charge, $Q$, and the voltage, $V$, on each of the capacitors.

Charges on $C_{1}$ and $C_{23}$ are equal $Q_{1}=Q_{23}=Q_{123}$ and equal to that on $C_{123}$

$$
\begin{gathered}
\mathrm{V}-\frac{\mathrm{Q}_{1}}{C_{1}}-\frac{\mathrm{Q}_{23}}{C_{23}}=\mathrm{V}-\frac{\mathrm{Q}_{123}}{C_{1}}+\frac{\mathrm{Q}_{123}}{C_{23}}=\mathrm{V}-\mathrm{Q}_{123}\left(\frac{1}{C_{1}}+\frac{1}{C_{23}}\right)=0 \\
\mathrm{Q}_{1}=\mathrm{Q}_{23}=\mathrm{V} C_{123}=(20 \mathrm{~V})(1.33 \mu \mathrm{~F})=26.6 \mu \mathrm{C}
\end{gathered}
$$

Voltages across $C_{1}$ and $C_{23}$ are not equal and sum to $V$

$$
V_{1}=\frac{Q_{123}}{C_{1}} \& V_{23}=\frac{Q_{123}}{C_{23}}
$$

$$
\left.\begin{array}{l}
\mathrm{V}_{1}=\frac{\mathrm{Q}_{123}}{C_{1}}=\frac{26.6 \mu \mathrm{C}}{2.0 \mu \mathrm{~F}}=13.3 \mathrm{~V} \\
\mathrm{~V}_{23}=\frac{\mathrm{Q}_{123}}{C_{23}}=\frac{26.6 \mu \mathrm{C}}{4.0 \mu \mathrm{~F}}=6.65 \mathrm{~V}
\end{array}\right\} \mathrm{V}_{1}+\mathrm{V}_{23}=\mathrm{V}
$$

Voltages across $C_{2}$ and $C_{3}$ are equal and equal to that on $C_{23}$

$$
V_{23}=V_{2}=V_{3}=6.65 \mathrm{~V}
$$

Charges on $C_{1}$ and $C_{23}$ are not equal and sum to $Q_{23}$
$Q_{123}=\frac{V_{23}}{C_{23}}=Q_{2}+Q_{3}$

$$
\left.\begin{array}{l}
Q_{2}=V_{23} C_{2}=(6.65 \mathrm{~V})(1 \mu \mathrm{~F})=6.65 \mu \mathrm{C} \\
\mathrm{Q}_{3}=\mathrm{V}_{23} C_{3}=(6.65 \mathrm{~V})(3 \mu \mathrm{~F})=19.95 \mu \mathrm{C}
\end{array}\right\} \quad \mathrm{Q}_{2}+\mathrm{Q}_{3}=\mathrm{Q}_{23}
$$

Summarize the capacitance, charge and voltage of each capacitor


