Special Relativity Worksheet

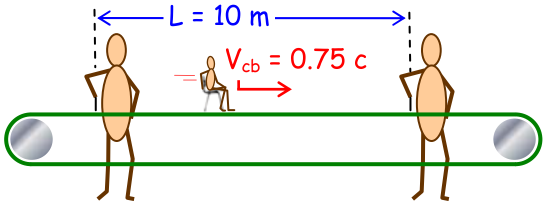
1. A relativistic conveyor belt (reference frame S') is moving at a speed v = 0.75c relative to a reference frame, S. Two observers in reference S, are standing beside the belt 10 m apart. They each paint a mark on the belt at exactly the same instant (as measured in S). (From Taylor, Zaffiratos and Dubson's *Modern Physics for Scientists and Engineers*.)

   a) What are the two events?

   b) In what reference frame are the two events at rest?

   c) find gamma

   d) find the distance between the marks as measured by the observers on the belt (in S')



a) The events are the marks on the belt.

b) The marks are at rest in frame S', that of the conveyor belt. Since those who mark the belt see the marks moving, they measure L = 10 m and L_0 is measured by the observer on the belt.

c) $$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{(0.75c)^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 1.511$$

$$L = \frac{L_0}{\gamma} \Rightarrow L_0 = \gamma L = (1.511)(10) = 15.11\text{ m}$$

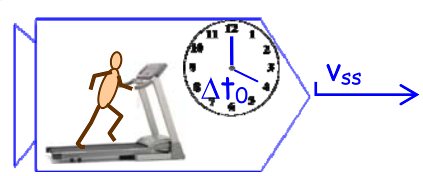
d) The observer on the belt sees the marks 15.11 m apart

2. A spaceship resting on Earth has a length of 35.2m. As it departs on a trip to another planet, it has a length of 30.5 m as measured by the Earth observers. The Earth observers also notice that one of the astronauts on the spaceship exercises for 22.2 min. (problem 61 from GRR)

   a) What is gamma? Think about what distance has the proper length.

   b) In which reference frame is the proper time for the exercises measured?

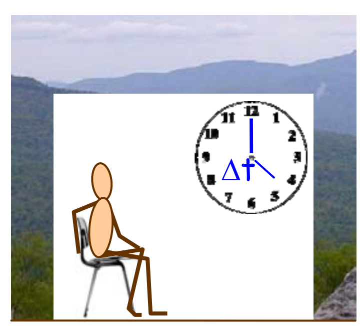
   c) How long would the astronaut herself say that she exercises?



a) L_0 is the length of the ship measured on Earth. Once it's moving, Earth observers measure its length as L:

$$L = \frac{L_0}{\gamma} \Rightarrow \gamma = \frac{L_0}{L} = \frac{35.2}{30.5} = 1.15$$

b) Δt_0 is the time measured on a clock that isn't moving so it's the time measured by the astronaut on her clock.

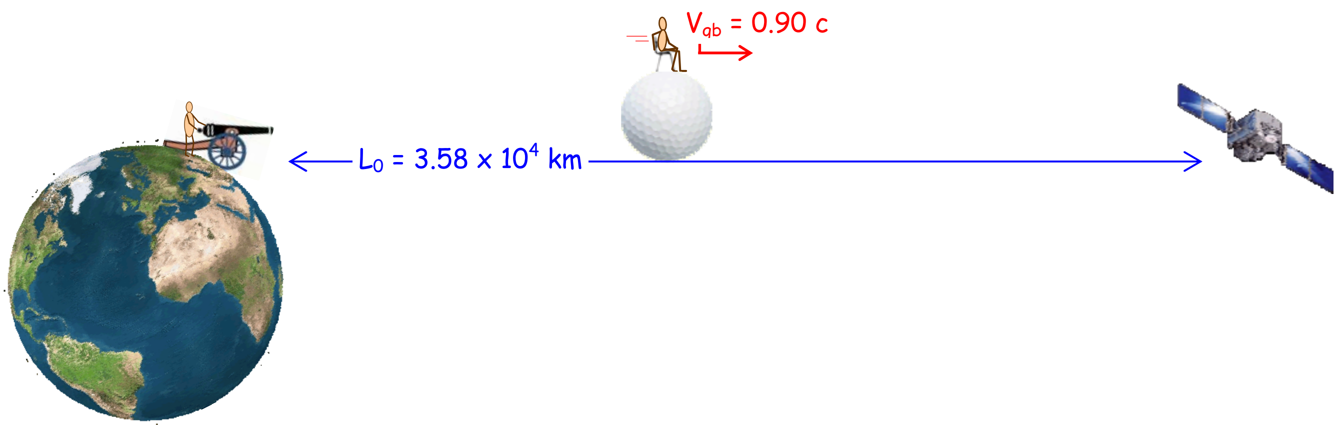


c)

$$\Delta t = \gamma \Delta t_0 \Rightarrow \Delta t_0 = \frac{\Delta t}{\gamma} = \frac{22.2}{1.15} = 19.3\text{ min}$$

3) A mechanism on earth used to shoot down geosynchronous satellites that house laser based weapons is finally perfected and propels golf balls at 0.9c. (from Thornton and Rex, Modern Physics for Scientists and Engineers)

a) How far will a detector riding with the golf ball initially measure the distance to the satellite? (Geosynchronous satellites are 3.58×10^4 km above Earth's surface.)
b) How long will it take the golf ball to make the journey to the satellite in Earth's frame?
c) How long will it take in the golf ball's frame?
d) Which reference frame has the proper time? Check yourself with a calculation.



a) L_0 is the distance to the satellite measured by Earth. The golf ball observer will measure L:

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{(0.90c)^2}{c^2}}} = 2.29 \qquad L = \frac{L_0}{\gamma} = \frac{3.58\times10^4 \text{ km}}{2.29} = 1.56\times10^4 \text{ km}$$

b) Use kinematics to find ball's travel time according to Earth:

$$x_E = v_E t_E \quad \Rightarrow \quad t_E = \frac{v_E}{x_E} = \frac{\left(3.58\times10^7\right)}{(0.90)\left(3\times10^8\right)} = 0.132 \text{ s}$$

c) Use kinematics to find ball's travel time according to the golf ball:

$$x_{gb} = v_{gb} t_{gb} \quad \Rightarrow \quad t_{gb} = \frac{x_{gb}}{v_{gb}} = \frac{\left(1.56\times10^7\right)}{(0.90)\left(3\times10^8\right)} = 0.578 \text{ s}$$

d) The golf ball observer's watch does not move according to the golf ball observer, so she measures Δt_0 and the Earth observer measures Δt. Use the time dilation equation to check this:

$$\Delta t = \gamma \Delta t_0 \quad \Rightarrow \quad t_E = \gamma t_{gb} = (2.29)(0.578) = 1.32 \text{ s}$$

Yep! It agrees with the other calculation of the time measured by Earth. Ta Da!