

Our goal is to determine the current through and voltage across each of the resistors in the circuit below. There is more than one way to approach this problem, but here we will use the idea of series and parallel circuits.

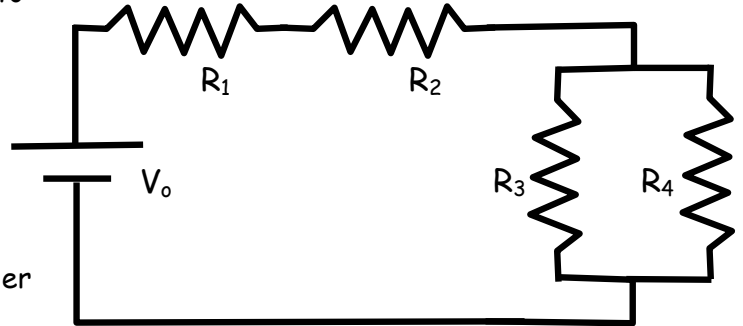
In this circuit, $V_0 = 10\text{ V}$, $R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$, $R_3 = 40\ \Omega$ and $R_4 = 60\ \Omega$

Let's begin by reducing our circuit to one resistor.

1. Look at the circuit and determine whether

there are any resistors in series. If there are, reduce them to one resistor called R_{eq1} , and sketch the new circuit below on the right. $R_{eq} = R_a + R_b + R_c$

$$\begin{aligned} R_{eq1} &= R_1 + R_2 \\ &= 10\ \Omega + 20\ \Omega \\ &= 30\ \Omega \end{aligned}$$



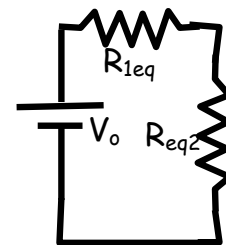
series

2. Look at the circuit and determine whether there are any resistors in parallel. If there are, reduce them to one resistor called R_{eq2} , and sketch the new

circuit on

$$\begin{aligned} \frac{1}{R_{eq2}} &= \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{R_3} \frac{R_4}{R_4} + \frac{1}{R_4} \frac{R_3}{R_3} = \frac{R_3 + R_4}{R_3 R_4} \\ R_{eq2} &= \frac{R_3 R_4}{R_3 + R_4} = \frac{40\ \Omega \cdot 60\ \Omega}{40\ \Omega + 60\ \Omega} = \frac{240\ \Omega}{100} = 24\ \Omega \end{aligned}$$

the right.

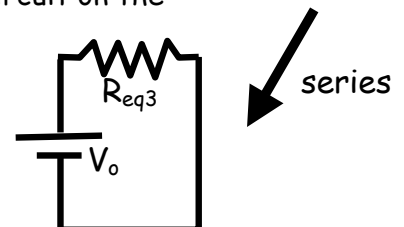


parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_a} + \frac{1}{R_b} + \frac{1}{R_c}$$

3. Look at the circuit and determine whether there are any resistors in series. If there are, reduce them to one resistor called R_{eq3} , and sketch the new circuit on the right.

$$\begin{aligned} R_{eq3} &= R_{1eq} + R_{2eq} \\ &= 30\ \Omega + 24\ \Omega \\ &= 54\ \Omega \end{aligned}$$



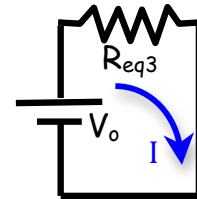
series

At this point we have one resistor, but you could continue this process if there were more ways to reduce the circuit.

4. Eventually we have worked our way down to one resistor (not all circuits can be reduced to a single element). Find the current through this resistor. $V = IR$

$$V = IR$$

$$I = \frac{V}{R} = \frac{10V}{54\Omega} = 0.185A$$



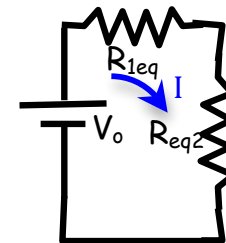
5. Now we can work our way backwards. We know the voltage across and the current through R_{eq3} . Since it was formed from the series combination of R_{eq2} and R_{eq1} , and since elements in series have the same current, we know the current through R_{eq2} and R_{eq1} . Since we know the current and the resistance, we can find the voltages V_{2eq} and V_{1eq} .

$$V = IR$$

$$V_{1eq} = IR_{eq1} = 0.185A \cdot 30\Omega = 5.56V$$

$$V = IR$$

$$V_{2eq} = IR_{eq2} = 0.185A \cdot 24\Omega = 4.44V$$



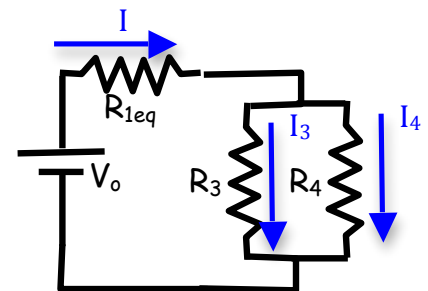
Note these add to 10 V, $V_{1eq} + V_{2eq} = 10V$

6. Since R_{eq2} was formed from the parallel combination of two resistors, and since we know that resistors in parallel have the same voltage, we now know the voltage across these two resistors. AND since we know their resistance, we can find the current through them.

$$I_3 = \frac{V_{eq2}}{R_3} = \frac{V_3}{R_3} = \frac{4.44V}{40\Omega} = 0.111A$$

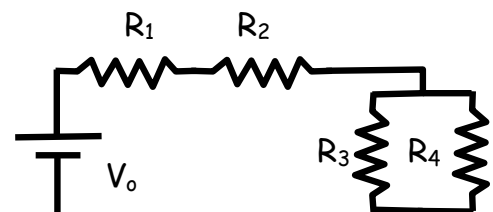
$$I_4 = \frac{V_{eq2}}{R_4} = \frac{V_4}{R_4} = \frac{4.44V}{60\Omega} = 0.074A$$

Note these
add to
0.185A
 $I = I_3 + I_4$



7. Finally, in part 4 we found the current that goes through R_{eq1} since this resistance is formed from the series resistance of two elements, and since elements in series have the same current, we know the current through those two resistors. Since we know the current and resistance, we can find the voltage.

$$V_1 = IR_1 = 0.185A \cdot 10\Omega = 1.85V$$



$$= IR_2 = 0.185 A \cdot 20 \Omega$$

$$= 3.70 V$$

Notice $V_1 + V_2 + V_{2eq} = 10V$

