HOMEWORK SET 7: HYDROGEN WAVE EQUATION

Due Monday, February 12, 2024

PROBLEMS FROM TZDII[1]

1) 8.44

2) 8.47

PROBLEM FROM AOD

3) The general 3-dimensional Schrödinger Equation in spherical cooridinates is (equation 8.49 with substitutions)

$$a_B = \frac{\hbar^2}{m_e k e^2} \qquad E = -\frac{E_R}{n^2} = -\frac{m_e\left(ke^2\right)^2}{2\hbar^2 n^2} = -\frac{ke^2}{2a_B n^2}$$

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}\left(r\psi\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\varphi^2} = \left[\frac{1}{a_B^2 n^2} - \frac{2}{a_B r}\right]\psi$$

a) Using the tables in the text, write out the separate solutions for $R_{2,1}(r)$, $\Theta_{1,-1}(\theta)$ and $\Phi_{-1}(\phi)$ then write out $\psi_{2,1,-1}(r,\theta,\phi)$ (COMBINE ALL THE CONSTANTS INTO THE SIMPLEST FORM. FOR PART B, LET $A = \frac{1}{8}\sqrt{\frac{1}{a_B^5\pi}}$)

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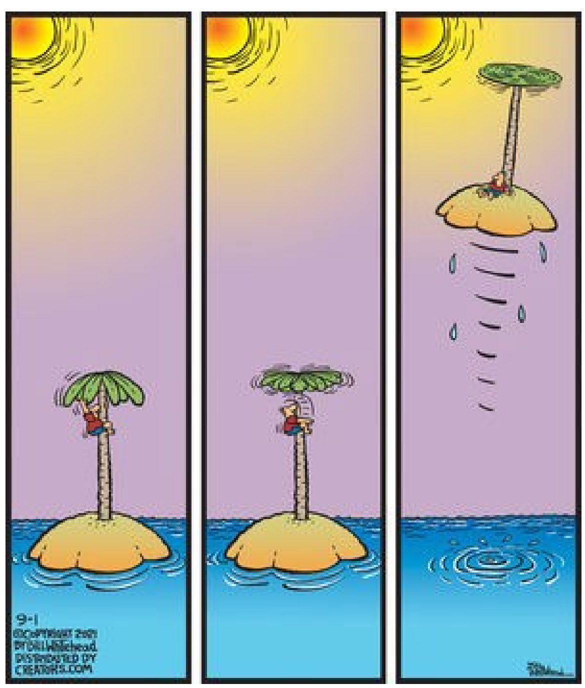
b) Show that the $\psi_{2,1,-1}(r,\theta,\phi)$ you've written down is a solution to the 3-D Schrödinger equation shown above. (HINT: EVALUATE EACH TERM ON THE LEFT SEPARATELY (EACH TERM WITH PARTIAL DERIVATIVES) SIMPLIFY THEM SEPARATELY, THEN ADD THEM TOGETHER. SUBSTITUTE ψ ON THE RIGHT, CANCEL COMMON TERMS AND DO THE ALGEBRA TO GET 1 = 1 OR 0 = 0)



[1] Taylor, Zafiratos, & Dubson, *Modern Physics for Scientists and Engineers*, 2nd Editon, Pearson, Prentice Hall, 2004