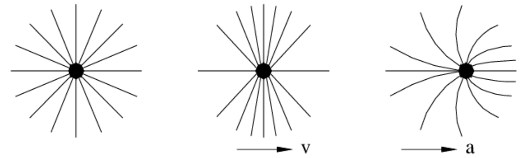
HOMEWORK SET 13: RADIATION FROM ACCELERATED CHARGES & SELECTION RULES
Due Friday, March 8, 2024

Problems adapted from TZDII[1]

Field lines of charges at various speeds



(a) At Rest. **(b)** Uniform Velocity. **(c)** Accelerated.

11.5) The formula for the power P radiated by an accelerating charge q,

$$P = \frac{2kq^2a^2}{3c^3} \qquad (1)$$

can be derived by the method of **dimensional analysis**. Since P would be expected to involve k, q, a (the acceleration), and c, we might reasonably guess that it should have the form

$$P = bk^\ell q^m a^n c^p \qquad (2)$$

where b is some dimensionless number of order 1 (perhaps something like 4π) and where ℓ, m, n, and p are unknown powers (and k is the Coulomb force-constant ≈ 9 x 10^9 N-m^2/C^2). By inserting their units into the five dimensional quantities in (2), you will get an equation that determines the unknown powers ℓ, m, n, and p. Show that you obtain the correct form (1), except that the dimensionless number b cannot be determined.

11.6 & 15) The result (1) shows that a classical, radiating atom would collapse very rapidly. To get a rough estimate of the time for a hydrogen atom to collapse completely,

a) Find the rate at which the radius r shrinks,

$$\frac{dr}{dt} = \frac{dr}{dE}\frac{dE}{dt} \qquad (3)$$

using dr/dE determined from the energy in a circular orbit[2],

$$E = -\frac{1}{2}\frac{ke^2}{r} \qquad (4)$$

and the power as the rate of energy loss, dE/dt = -P. Find dr/dt when r = a_0. [Solve (4) for r, write out dr/dE, then substitute E from (4). Determine dE/dt from (1) with the acceleration of the electron found from F_E/m. Substitute these into (3) to obtain an expression for and calculate the rate at which the electron falls into the nucleus, $\frac{dr}{dt} = -\frac{4(ke^2)c}{3(mc^2)^2 a_0^2}$]

b) Making the admittedly crude approximation that dr/dt remains constant (it falls in at constant speed giving dr/dt = -A), estimate how long the electron takes to spiral in from r = a_0 to r = 0. [Solve the differential equation dr/dt = -A by separation of variables, integrating dr from a_0 to 0 to determine the time to collapse. It should be on the order of 10^{-11} seconds.]

Extra Credit

c) For a more realistic estimate, use (3) to find dr/dt, the rate at which the orbit shrinks as a function of r, and find the time for the classical atom to collapse entirely by evaluating the integral in

$$\frac{dr}{dt} = \frac{dr}{dt} \quad \Rightarrow \quad dt = \frac{dr}{dr/dt} \quad \Rightarrow \quad t_{collapse} = \int_{a_0}^{0} \frac{dr}{dr/dt}$$

[Use the espression for dr/dt with r instead of a_0. You should get $t_{collapse} = \left(\frac{mc^2}{2ke^2}\right)^2 \frac{a_0^3}{c}$ which has a slightly smaller value than you got in part b.]

---

[1] Taylor, Zafiratos, & Dubson, *Modern Physics for Scientists and Engineers*, 2nd Editon, Pearson, Prentice Hall, 2004

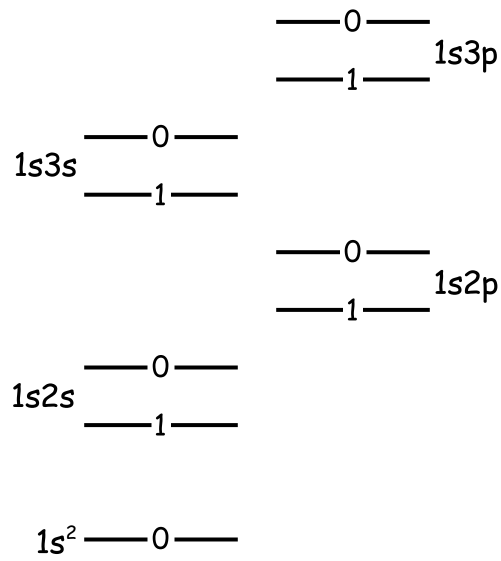
[2] For an electron orbiting a proton NSL gives $mv^2/r = ke^2/r^2$, so the kinetic energy, $K = mv^2/2 = ke^2/2r$ (half the potential energy, a result of the Virial Theorem. Recalling the potential energy, $U(r) = -ke^2/r$, the total energy is $E = K + U(r) = -ke^2/2r$.

11.25) Figure 11.23 shows some of the lowest energy levels of the He atom. They are labeled by their configuration (for example, 1s2p means that the atom has one electron in the 1s level and one in the 2p level). The energy depends somewhat on the orientation of the two electrons' spins: If the spins are antiparallel, the total spin is zero (quantum number $s_{tot} = 0$); if the spins are parallel, the total spin has $s_{tot} = 1$. For a given configuration, the state with $s_{tot} = 1$ has slightly lower energy.

a) Explain why the 1s configuration has only $s_{tot} = 0$.

b) There is a selection rule $\Delta s_{tot} = 0$, that is, transitions in which s_{tot} changes are forbidden. Indicate all allowed transitions on the energy-level diagram. (Copy or neatly re-draw the figure into your solution. Don't forget the selection rule $\Delta\ell = +1$.)

c) Which excited levels would you expect to be metastable?



11.29) Consider two energy levels of the helium atom, in both of which the two electrons' spins are antiparallel (so that the total spin is zero, and the spins can be ignored) and one of the electrons is in the lowest (1s) orbital. In the upper level the second electron in in a d, $\ell = 2$ orbital from which it can transition down to a lower level f, $\ell = 3$ orbital. The atom is placed in a magnetic field, and the upper level splits into seven equally spaced sublevels and the lower into five sublevels (with the same spacing).

a) Sketch the resulting levels. (The UPPPER level has $\ell = 2$!)

b) If we ignore selection rules, there are, in principle, 35 different possible transitions from the upper ($\ell = 2$) level to the lower ($\ell = 3$) level. Using the energy level splitting in a magnetic field (ignoring any energy difference due to changes in n)

$$\Delta E_{\ell,m} = \left(E_{\ell=2} + m_{\ell=2}\mu_B B\right) - \left(E_{\ell=3} + m_{\ell=3}\mu_B B\right) = \left(E_{\ell=2} - E_{\ell=3}\right) + \left(m_{\ell=2} - m_{\ell=3}\right)\mu_B B$$

Giving the

$$\Delta E_{\ell,m} = \Delta E_{\ell=3 \text{ to } \ell=2} + \left(m_{\ell=2} - m_{\ell=3}\right)\mu_B B \qquad (1)$$

Show that because the sublevels all have the same spacing, there are actually only eleven distinct energy differences.

c) The selection rules for these transitions are $\Delta\ell = +1$ and $\Delta m_\ell = 0$ or ± 1; that is, only transitions that satisfy these rules are allowed. Draw all of the allowed transitions on your energy level diagram.

d) How many distinct photon frequencies will result from allowed transitions between the two levels? This is the normal Zeeman effect.