Homework Set 17: Statistical Mecahnics
Due Wednesday, March 27, 2024

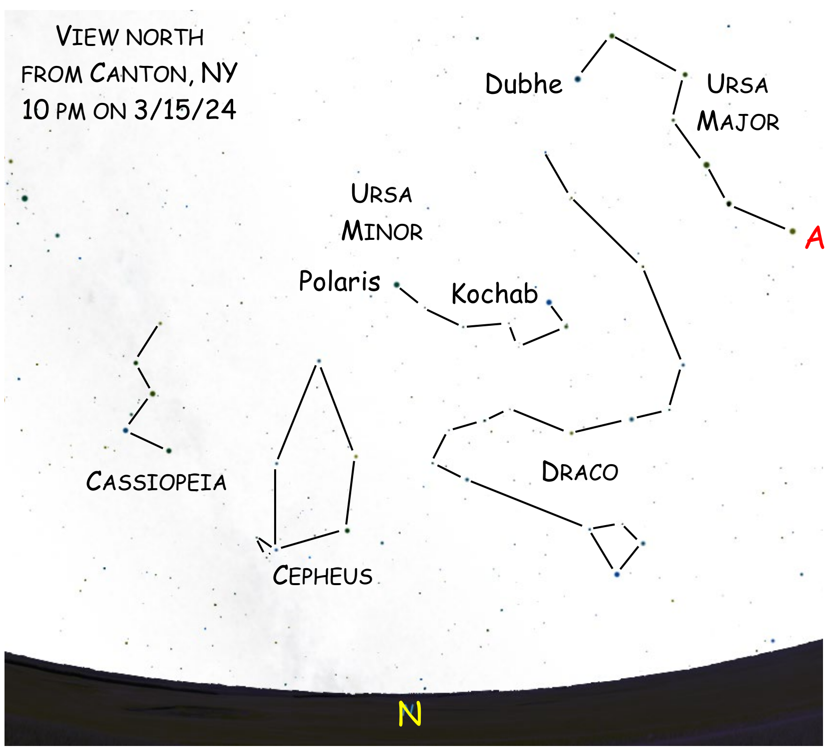
Problem from Serway, Moses & Moyer, *Modern Physics* (2nd ed.)[1]

1) a) Find the populations of the first and second excited states relative to the ground state for atomic hydrogen at 300K assuming that it obeys Maxwell-Boltzmann statistics.

b) Assuming Maxwell-Boltzmann statistics, find the populations of the first and second excited states relative to the ground state for atomic hydrogen heated to 15,500 K in Alkaid (η UMa, "Eta Ursae Majoris", the star at the end of the handle of the Big Dipper).

c) for the populations in **b**, given that the Balmer absorption lines are upward transitions from the first excited state and the Paschen absorption lines are upward transitions from the second excited state, which absorption lines should be stronger, Balmer or Paschen (for equal numbers of incoming photons with appropriate energies)? Hint: Partlicles are more likely to move from a more-populated state to a less-populated state than vise versa.)

Answers: a) $\left.\frac{n_2}{n_1}\right|_{300K} = 4e^{-394} \approx 0, \ \left.\frac{n_3}{n_1}\right|_{300K} = 9e^{-468} \approx 0$ b) $\left.\frac{n_2}{n_1}\right|_{15,000K} = 4e^{-7.63} = 0.0019, \ \left.\frac{n_3}{n_1}\right|_{15,000K} = 9e^{-9.06} = 0.001$



[1] Serway, Moses, & Moyer, Modern Physics, 2nd Editon, Saunders, Harcort Brace College Publishers, 1997

PROBLEMS FROM OR AFTER THORNTON & REX (TREX 3RD ED.)[2]

9.15) As written in TRex.

Hints: The M-B Energy Distribution is

$$F_{MB}(E) = \frac{8\pi}{\sqrt{2m^3}}\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-E/kT}\sqrt{E} \qquad (9.26)$$

The mean energy is then

$$\bar{E} = \int_0^\infty EF(E)\,dE$$

The CRC gives the integral (#369) as

369. $\int_0^\infty x^n e^{-ax}\,dx$

$$= \begin{cases} \dfrac{\Gamma(n+1)}{a^{n+1}}, & (n>-1,\ a>0) \\ \text{or} & \\ \dfrac{n!}{a^{n+1}}, & (a>0,\ n \text{ positive integer}) \end{cases}$$

9.2 Gamma Function (Generalized Factorial Function)

The gamma function, denoted $\Gamma(x)$, is defined by

$$\Gamma(x) = \int_0^\infty e^{-t}t^{x-1}dt, \quad x>0$$

- *Properties*

$\boxed{\Gamma(x+1) = x\Gamma(x), \quad x>0}$

$\Gamma(1) = 1$

$\Gamma(n+1) = n\Gamma(n) = n! \quad (n = 1, 2, 3, \ldots)$

$\Gamma(x)\Gamma(1-x) = \pi/\sin\pi x$

$\boxed{\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}}$

$2^{2x-1}\Gamma(x)\Gamma\left(x+\frac{1}{2}\right) = \sqrt{\pi}\Gamma(2x)$

Where the top form applies to your integral (with $E^{3/2}$). The Gamma (Γ) functions are explained on CRC pp. 128-129. The properties that apply here are circled in green on the image above [don't concern yourself with the integral in the explanation unless it interests you. You should get the integral itself (not multiplied by any of the constants) to be

$$(kT)^{5/2}\left(\frac{3\sqrt{\pi}}{4}\right)$$

9.17) As written in TRex. Answer = 6,761K

[2] Thornton & Rex, Modern Physics for Scientists and Engineers, 2nd Editon, Saunders, Harcort Brace College Publishers, 2000