Electron Contribution to Heat Capacity: 
Fermi-Dirac Energy Distributions

The energy distribution of particles for a variety of temperatures is shown with $T = 0$ being the step function and curves for higher temperatures getting successively smoother.

Looking at just the $T = 0$ and $T \approx 5000K$ curves, the number of electrons excited out of their ground state can be approximated by triangles as shown:

The number of excited electrons is the area of the triangle and the energy of the collection of particles is the number times the average change in energy and the heat capacity is the derivative

\[
U_{EX} = \left[ \frac{1}{2} \left( \frac{3}{4} \frac{N}{E_F} \right) (\alpha kT) \right] \left( E_F + \frac{1}{3} \alpha kT \right) - \left( E_F - \frac{1}{3} \alpha kT \right) = \frac{1}{4} \frac{N(\alpha kT)^2}{E_F}
\]

\[
C_{V,e} = \frac{\pi^2}{2} N A k \frac{T}{T_F} = \frac{\pi^2}{2} R \frac{T}{T_F}
\]