16.25) Estimate the potential energy of an $\alpha$ in the center of the $^{27}\text{Al}$ nucleus. Compare answer with the minimum KE of $^{13}\text{C}$ in a box $k_{\text{min}} = 180\text{MeV}$ (16.10)

Using 16.49 (from the hint)

$$V(r) = \frac{kQ}{2\pi} \left( 3 \cdot \frac{r^2}{R^2} \right) \quad (r \leq R)$$

The charge on the $\alpha$ is $Q = (2 + 1)e = 14e$

The radius of the nucleus is $R = R_0 A^{1/3} = (1.07)(27)^{1/3} = 3.21\text{ fm}$

At $r = 0$ the potential energy of the $\alpha$ is

$$V(0) = -\frac{8.99 \times 10^9 (14)(1.6 \times 10^{-19})^2}{2(3.21 \times 10^{-15})} (3 - 0)$$

$$= 1.5 \times 10^{-12} J = -9.4\text{ MeV} = \text{PE}$$

This is much less than the minimum KE of the electron in a box, so it would escape.

16.30) a) Write down and sketch $U_{\text{Coul}}(r)$ for a proton in a nucleus approximated by a uniform sphere of charge $(Z-1)e$ and radius $R$.

b) Prove $U_{\text{Coul}}(0) = 1.5 U_{\text{Coul}}(R)$.

c) Setting $R = R_0 A^{1/3}$, estimate $U_{\text{Coul}}(0)$ for a $^{4}\text{He}$ and one in $^{209}$U.

a)

$$U_{\text{Coul}}(r) = 8V(r) = (Z-1) \frac{k e^2}{r} \quad (r > R)$$

$$= (Z-1) \frac{k e^2}{2R} \left( 3 - \frac{r^3}{R^3} \right) \quad (r \leq R)$$

See plot attached.

b)

$$U_{\text{Coul}}(0) = 3(Z-1) \frac{k e^2}{2R}$$

$$U_{\text{Coul}}(R) = (Z-1) \frac{k e^2}{2R} \left( 3 - \frac{R^3}{R^3} \right) = 2(Z-1) \frac{k e^2}{2R}$$

$$\Rightarrow \frac{U_{\text{Coul}}(0)}{U_{\text{Coul}}(R)} = \frac{3}{2} = 1.5$$
16.30) Continued

\( c) \) \( U_{\text{atom}} = R - R_0 A^{1/3} \)

\[ U_{\text{core}}(0) = \frac{3(2-1) k e^2}{2R} = \frac{3(2-1) k e^2}{2 R_0 A^{1/3}} \]

For \( ^4 \text{He} \)

\[ U_{\text{He}}(0) = \frac{3(2-1)(1.44 \text{ MeV} \cdot \text{fm})}{2(1.07 \text{ fm})(4)^{1/3}} = 1.27 \text{ MeV} = U_{\text{He}}(0) \]

For \( ^{238} \text{U} \)

\[ U_{\text{U}}(0) = \frac{3(92-1)(1.44 \text{ MeV} \cdot \text{fm})}{2(1.07 \text{ fm})(238)^{1/3}} = 29.6 \text{ MeV} = U_{\text{U}}(0) \]