17.23. If the mass of the neutrino is neglected, the kinetic energy released in $\beta^-$ decay is

$$\Delta E = \mathcal{K} = \Delta M_{\alpha}^2 = m_{\text{atom}}(Z,N) - m_{\text{atom}}(Z-1,N-1) \to c^2 \quad (17.72)$$

And $\mathcal{K}$ must be positive for the decay to be possible.

(a) Find an expression for $\mathcal{K}$ released in $\beta^-$

$$(Z,N) \to (Z-1,N+1) + e^+ + \gamma$$

And show that for this to be possible

$$m_{\text{atom}}(Z,N) \geq m_{\text{atom}}(Z-1,N+1) + 2m_e \quad (17.74)$$

(b) Do the same for $e^-$ capture

$$(Z,N) \to (Z-1,N+1) + \gamma$$

(c) Show that $^{231}U$ can decay to $^{231}Pa$ by $e^-$ capture, but not $\beta^+$

(a) For $\beta^+$ decay

Losing a positron = gaining an $e^-$'s mass

$$\beta^+ \to n^0 + e^+$$

$$m_{\beta^+} \to m_{n^0} + m_{e^-} \Rightarrow \text{Doesn't occur spontaneously}$$

So, for a nucleus

$$(Z,N) \to (Z-1,N+1) + e^+ + \gamma$$

The masses are

$$m_{\text{nucl}}(Z,N) \leq m_{\text{nucl}}(Z-1,N+1) + m_{e^-}$$

For atoms, the $Z e^-$ must be accounted for

$$m_{\text{nucl}}(Z,N) + 2m_e \leq m_{\text{nucl}}(Z-1,N+1) + (Z-1)m_e + m_{e^-} + m_{e^-}$$

Thus

$$m_{\text{atom}}(Z,N) \leq m_{\text{atom}}(Z-1,N+1) + 2m_e \quad (17.74)$$

QED
17.23 CONTINUED

b) For e\(^{-}\) capture

\[(Z, N) + e^{-} \rightarrow (Z-1, N+1) + 2\]

\[m_{\text{nucl}}(Z, N) + m_e \geq m_{\text{nucl}}(Z-1, N+1)\]

Change to atoms with 2 e\(-s\):

\[m_{\text{nucl}}(Z, N) + 2m_e + m_e \geq m_{\text{nucl}}(Z-1, N+1) + (2-1)m_e + m_e\]

Thus

\[m_{\text{atom}}(Z, N) + m_e \geq m_{\text{atom}}(Z-1, N+1) + m_e\]

Thus

\[m_{\text{atom}}(Z, N) \geq m_{\text{atom}}(Z-1, N+1)\]

which is 17.84 without the \(2m_e\).

c) Show that \(^{231}\text{U}\) can decay by e\(^{-}\) capture but not \(\beta^+\) emission.

\(^{231}\text{U}\) has 231.03626 u

\(^{231}\text{Pa}\) has 231.03588 u

For e\(^{-}\): \(m_U \geq m_{\text{Pa}}\)

Since

\[m_U - m_{\text{Pa}} = 3.8 \times 10^{-4}\text{ u} \Rightarrow \text{e}^{-}\text{ capture possible}\]

For e\(^{-}\): \(m_U - m_{\text{Pa}} > 2m_e\)

\[\Rightarrow 3.8 \times 10^{-4} > 2m_e > (2)(5.485799110 \times 10^{-4})\]

\[\Rightarrow 3.8 \times 10^{-4} > 10.97 \times 10^{-4}\]

so \(\beta^+\) is impossible for \(^{231}\text{U}\)!

Cool!