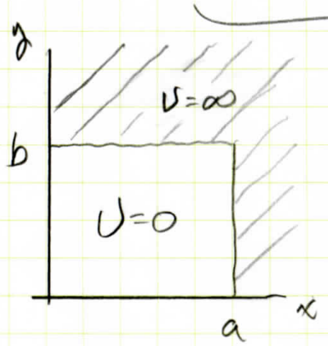
8.9) A PARTICLE IS IN A 2-D BOX WITH $0<x<a$, $0<y<b$. FIND THE ALLOWED ENERGY LEVELS,

$$E_{n_x,n_y} = \frac{\hbar^2\pi^2}{2M}\left(\frac{n_x^2}{a^2} + \frac{n_y^2}{b^2}\right)$$



THE SCHRODINGER EQUATION IN 2-D IS

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = \frac{2M}{\hbar^2}\left[U(x) - E\right]\psi$$

HERE

$$U = \begin{cases} 0 & 0<x<a,\ 0<y<b \\ \infty & \text{ELSEWHERE} \end{cases}$$

THUS, IN THE BOX

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = -\frac{2M}{\hbar} E\psi \qquad (8.6)$$

TO SOLVE THIS, USE SEPARATION OF VARIABLES

ASSUME

$$\psi(x,y) = X(x)Y(y)$$

THEN

$$Y(y)X''(x) + X(x)Y''(y) = -\frac{2M}{\hbar} E X(x)Y(y)$$

DIVIDING THROUGH BY $X(x)Y(y)$ GIVES

$$\frac{X''(x)}{X(x)} + \frac{Y''(y)}{Y(y)} = \frac{2M}{\hbar}E$$

SINCE THE SUM OF FUNCTIONS OF DIFFERENT VARIABLES IS A CONSTANT, EACH MUST ALSO BE A CONSTANT, THUS

$$X''(x) = -k_x^2 X(x) \qquad Y(y) = -k_y^2 Y(y)$$

$$\Rightarrow \quad X(x) = A\sin k_x x \qquad Y(y) = B\sin k_y y$$

APPLY THE BOUNDARY CONDITIONS

$$X(0) = X(a) = 0 \quad \Rightarrow \quad k_x = \frac{n\pi}{a}$$

$$Y(0) = Y(b) = 0 \quad \Rightarrow \quad k_y = \frac{n\pi}{b}$$

8.9) CONTINUED

THEN

$$X(x) = A\sin\left(\frac{n_x \pi x}{a}\right) \qquad Y(y) = B\sin\left(\frac{n_y \pi y}{b}\right)$$

THE COMPLETE WAVE FUNCTION IS THUS

$$\psi(x,y) = AB\sin\left(\frac{n_x \pi x}{a}\right)\sin\left(\frac{n_y \pi y}{b}\right).$$

REPLACING THIS IN THE DE & DIVIDING BY $\psi(x,y)$ GIVES

$$\frac{-\frac{A n^2 \pi^2}{a^2}\sin\left(\frac{n_x \pi x}{a}\right)}{A\sin\left(\frac{n \pi x}{a}\right)} + \frac{-\frac{B n^2 \pi^2}{b^2}\sin\left(\frac{n_y \pi y}{b}\right)}{B\sin\left(\frac{n \pi y}{b}\right)} = -\frac{2ME}{\hbar}$$

THUS $$-\frac{n_x^2 \pi^2}{a^2} - \frac{n_y^2 \pi^2}{b^2} = \frac{2ME}{\hbar^2}$$

AND $$\boxed{E = \frac{\hbar^2 \pi^2}{2M}\left(\frac{n_x^2}{a^2} + \frac{n_y^2}{b^2}\right)}$$ QED!