8.18) a) USE $x = r\cos\phi$ AND $y = r\sin\phi$ (8.104) TO EVALUATE $\frac{\partial x}{\partial r}$, $\frac{\partial y}{\partial r}$, $\frac{\partial x}{\partial \phi}$ AND $\frac{\partial y}{\partial \phi}$

b) IF $f = e^{\sqrt{x^2+y^2}}$, USE $\frac{\partial f}{\partial r} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial r}$ TO FIND $\frac{\partial f}{\partial r}$.

c) WHAT IS $\frac{\partial f}{\partial \phi}$?

d) BY NOTICING $r = \sqrt{x^2+y^2}$ AND $f = e^r$, EVALUATE $\frac{\partial f}{\partial r}$ AND $\frac{\partial f}{\partial \phi}$ DIRECTLY AND CHECK YOUR ANSWERS IN b & c.

a) $\frac{\partial x}{\partial r} = \cos\phi$, $\frac{\partial x}{\partial \phi} = -r\sin\phi$

$\frac{\partial y}{\partial r} = \sin\phi$, $\frac{\partial y}{\partial \phi} = r\cos\phi$

b) $\frac{\partial f}{\partial r} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial r}$, SO FOR $f = e^{\sqrt{x^2+y^2}}$

$$\frac{\partial f}{\partial r} = \frac{1}{2\sqrt{x^2+y^2}}\left[2x(\cos\phi) + 2y(\sin\phi)\right] e^{\sqrt{x^2+y^2}}$$

$$\frac{\partial f}{\partial r} = \frac{x\cos\phi + y\sin\phi}{\sqrt{x^2+y^2}}\, e^{\sqrt{x^2+y^2}}$$

REPLACING $x = r\cos\phi$, $y = r\sin\phi$ AND $r = \sqrt{x^2+y^2}$ GIVES

$$\frac{\partial f}{\partial r} = \frac{r\cos^2\phi + r\sin^2\phi}{r}\, e^r$$

$$\Rightarrow \boxed{\frac{\partial f}{\partial r} = e^r}$$

c) $\frac{\partial f}{\partial \phi} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial \phi} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial \phi}$

$$\frac{\partial f}{\partial \phi} = \frac{1}{2\sqrt{x^2+y^2}}\left[2x(-r\sin\phi) + 2y(r\cos\phi)\right] e^{\sqrt{x^2+y^2}}$$

NOTING $r\sin\phi = y$ AND $r\cos\phi = x$

$$\frac{\partial f}{\partial \phi} = \frac{1}{\sqrt{x^2+y^2}}\left[-xy + xy\right]^0 e^{\sqrt{x^2+y^2}} = \boxed{0 = \frac{\partial f}{\partial \phi}}$$

d) FOR $f = e^r$

$\frac{\partial f}{\partial r} = e^r$ AND $\frac{\partial f}{\partial \phi} = 0 \Rightarrow$ b & c ARE CORRECT