2) STARTING WITH THE SCHRÖDINGER EQUATION (T. 6.16)

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(r\psi) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)\psi + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\psi = -\frac{2M}{\hbar^2}\left[E-V(r)\right]$$

SUBSTITUTE $\psi = R\Theta\Phi$, MULTIPLY BY $\frac{r^2\sin^2\theta}{R\Theta\Phi}$ AND SEPARATE VARIABLES TO DERIVE ORDINARY D.E.'S IN EACH VARIABLE

SUBSTITUTE $\psi = R\Theta\Phi$

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(rR\Theta\Phi) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)R\Theta\Phi +$$
$$+\frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}R\Theta\Phi = -\frac{2M}{\hbar^2}\left[E-V(r)\right]R\Theta\Phi$$

FACTOR OUT THE FUNCTIONS TREATED AS CONSTANTS BY THE PARTIAL DERIVATIVES

$$\frac{\Theta\Phi}{r}\frac{\partial^2}{\partial r^2}(rR) + \frac{R\Phi}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Theta}{\partial\theta}\right) +$$
$$+\frac{R\Theta}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\Phi = -\frac{2M}{\hbar^2}\left[E-V(r)\right]R\Theta\Phi$$

NOW MULTIPLY BY $\frac{r^2\sin^2\theta}{R\Theta\Phi}$

$$\frac{r\sin^2\theta}{R}\frac{\partial^2}{\partial r^2}(rR) + \frac{\sin\theta}{\Theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Theta}{\partial\theta}\right) +$$
$$+\frac{1}{\Phi}\frac{\partial^2\Phi}{\partial\phi^2} = -\frac{2M}{\hbar^2}\left[E-V(r)\right]r^2\sin^2\theta$$

b) ISOLATE THE ϕ TERM AND SET IT EQUAL TO $-m^2$

$$\frac{1}{\Phi}\frac{\partial^2\Phi}{\partial\phi^2} = -m^2 = -\frac{2Mr^2\sin^2\theta}{\hbar^2}\left[E-V(r)\right] - \frac{r\sin^2\theta}{R}\frac{\partial^2}{\partial r^2}(rR) - \frac{\sin\theta}{\Theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Theta}{\partial\theta}\right)$$

GIVING

$$\frac{\partial^2\Phi}{\partial\phi^2} = -m^2\Phi \Rightarrow \boxed{\frac{d^2\Phi}{d\phi^2} = -m^2\Phi}$$

SINCE ϕ IS THE ONLY VARIABLE!

2) CONTINUED

c) SEPARATE r & θ TERMS IN

$$m^2 = \frac{2Mr^2\sin^2\theta}{\hbar^2}\left[E-V(r)\right] + \frac{r\sin^2\theta}{R}\frac{\partial^2}{\partial r^2}(rR) + \frac{\sin\theta}{\Theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Theta}{\partial\theta}\right)$$

AND DIVIDE THROUGH BY $\sin^2\theta$ TO GET

$$-\frac{r}{R}\frac{\partial^2}{\partial r^2}(rR) - \frac{2Mr^2}{\hbar^2}\left[E-V(r)\right] = \frac{-m^2}{\sin^2\theta} + \frac{1}{\Theta\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Theta}{\partial\theta}\right)$$

SINCE EACH SIDE IS A FUNCTION OF A DIFFERENT VARIABLE, BOTH MUST EQUAL A CONSTANT, $-\lambda$, THUS

$$-\lambda = \frac{-m^2}{\sin^2\theta} + \frac{1}{\Theta\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Theta}{\partial\theta}\right)$$

$$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Theta}{\partial\theta}\right) + \left(\lambda - \frac{m^2}{\sin^2\theta}\right)\Theta = 0$$

THAT IS A FUNCTION OF θ ONLY SO IT BECOMES

$$\boxed{\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right) + \left(\lambda - \frac{m^2}{\sin^2\theta}\right)\Theta = 0}$$ QED!

THE r EQUATION IS NOW

$$-\frac{r}{R}\frac{\partial^2}{\partial r^2}(rR) - \frac{2Mr^2}{\hbar^2}\left[E-V(r)\right] = -\lambda$$

$$\frac{\partial^2}{\partial r^2}(rR) = -\frac{2M}{\hbar^2}\left[E-V(r)\right](rR) + \left(\frac{\lambda}{r^2}\right)(rR)$$

THAT IS A FUNCTION OF r ONLY GIVING

$$\boxed{\frac{d^2}{dr^2}(rR) = -\frac{2M}{\hbar}\left[E-V(r)\right](rR) + \frac{\lambda}{r^2}(rR)}$$ QED!