8.39) a) VERIFY THAT

$$\frac{d^2}{dr^2}(rR) = \frac{2m_e}{\hbar^2}\left[-\frac{ke^2}{r} + \frac{E_B}{n^2}\right](rR) \qquad (8.79)$$

CAN BE WRITTEN AS

$$\frac{d^2}{dr^2}(rR) = \left(\frac{1}{n^2 a_B^2} - \frac{2}{a_B r}\right)(rR) \qquad (8.107)$$

b) FOR $n=1$, PROVE $R_{1s} = e^{-r/a_B}$ IS A SOLUTION.

a) USING DEFINITIONS FROM ~~THE~~ BOHR ATOM,

$$a_B = \frac{\hbar^2}{m_e k e^2} \qquad \text{AND} \qquad E_B = \frac{ke^2}{2a_B}$$

IN 8.79 GIVES

$$\frac{d^2}{dr^2}(rR) = \left[-\frac{2m_e ke^2}{r\hbar^2} + \frac{\cancel{2}m_e}{\hbar^2 n^2}\left(\frac{ke^2}{\cancel{2}a_B}\right)\right](rR)$$

$$= \left[-\frac{2}{r}\left(\frac{m_e ke^2}{\hbar^2}\right) + \frac{1}{n^2}\left(\frac{m_e ke^2}{\hbar^2}\right)\frac{1}{a_B}\right](rR)$$

$$= \left[-\frac{2}{a_B r} + \frac{1}{n^2 a_B^2}\right](rR)$$

$$\boxed{\frac{d^2}{dr^2}(rR) = \left(\frac{1}{n^2 a_B^2} - \frac{2}{a_B r}\right)(rR)} \quad \text{QED!} \qquad (8.107)$$

b) FOR $n=1$, $R_{1s} = e^{-r/a_B}$

$$\frac{d^2}{dr^2}(rR) = \frac{d^2}{dr^2}\left(re^{-r/a_B}\right) = \frac{d}{dr}\left[\left(1-\frac{r}{a_B}\right)e^{-r/a_B}\right]$$

$$= \left[-\frac{1}{a_B} + \left(1-\frac{r}{a_B}\right)\left(\frac{-1}{a_B}\right)\right]e^{-r/a_B}$$

$$\frac{d^2}{dr^2}(rR) = \left(\frac{r}{a_B^2} - \frac{2}{a_B}\right)e^{-r/a_B} \qquad (1)$$

$\longrightarrow$

8.39) CONTINUED

THE RIGHT-HAND SIDE OF (8.107) FOR $n=1$ AND $R(r)=e^{-r/a_B}$

$$\left(\frac{1}{a_B^2}-\frac{2}{a_B r}\right)\left(r e^{-r/a_B}\right)=\left(\frac{r}{a_B^2}-\frac{2}{a_B}\right)e^{-r/a_B}$$

SETTING ~~THIS~~ EQUAL TO THE LEFT-HAND SIDE, (1)

$$\cancel{\left(\frac{r}{a_B^2}-\frac{2}{a_B}\right)e^{-r/a_B}}=\cancel{\left(\frac{r}{a_B^2}-\frac{2}{a_B}\right)e^{-r/a_B}}$$

$$1=1 \quad \text{QED!}$$

YES! $R(r)=e^{-r/a_B}$ IS A SOLUTION!

... AND ALL IS RIGHT WITH THE WORLD!

... YEAH, DON'T I WISH!