9.21) THE FINE STRUCTURE OF H IS DUE TO THE $\vec{B}$ FIELD EACH PARTICLE CREATES FOR THE OTHER

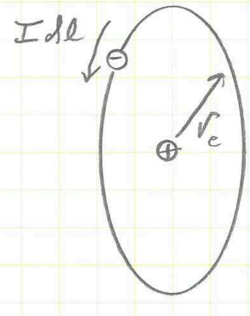
a) FOR $e^-$ AND $p^+$ AS CLASSICAL PARTICLES ORBITING EACH OTHER, USE BIOT-SAVART TO FIND $\vec{B}$

$$dB = \frac{\mu_0}{4\pi}\left|\frac{I\,d\vec{l}\times\hat{r}}{r^2}\right|$$

b) FIND I TO SHOW

$$B_{FS} = \frac{\mu_0}{4\pi}\frac{eL}{m_e r_c^2}$$

c) USE $L_{2p} = 2\hbar$ AND $r_{2p} = 4a_0$ TO FIND B AND $\Delta E_{UP-DOWN}$.




a) APPLY BIOT-SAVART

$$dB = \frac{\mu_0}{4\pi}\left|\frac{I\,d\vec{l}\times\hat{r}}{r^2}\right| = \frac{\mu_0}{4\pi}\frac{I\,dl}{r^2}$$

INTEGRATING $dl$ FROM 0 TO $2\pi r$ GIVES

$$B_{FS} = \frac{\mu_0 I}{4\pi}\int_0^{2\pi r}\frac{dl}{r^2} = \frac{\mu_0 I}{\cancel{4\pi}}\left(\frac{\cancel{2\pi r}}{r^{\cancel{2}}}\right)$$

$$\boxed{B_{FS} = \frac{\mu_0 I}{2r}} = \text{FIELD AT THE CENTER OF A CURRENT LOOP}$$

b) FIND THE CURRENT FOR AN ORBITING CHARGE, e ($e^-$ "SEES" THE PROTON ORBITING IT.)

$$I = \frac{\text{CHARGE}}{\text{TIME}} = \frac{e}{T}$$

THE PERIOD IS FOUND FROM THE VELOCITY

$$v = \frac{\text{DISTANCE}}{\text{TIME}} = \frac{2\pi r}{T} \Rightarrow T = \frac{2\pi r}{v}$$

THE ANGULAR MOMENTUM GIVES $v$

$$|\vec{L}| = |\vec{r}\times\vec{p}| = rmv \Rightarrow v = \frac{L}{rm_e}$$

$$\Rightarrow I = e\left(\frac{v}{2\pi r}\right) = \left(\frac{e}{2\pi r}\right)\left(\frac{L}{rm_e}\right) = \frac{eL}{2\pi m_e r^2}$$

$$B_{FS} = \frac{\mu_0}{2r}\left(\frac{eL}{2\pi m_e r^3}\right) = \boxed{\frac{\mu_0}{4\pi}\frac{eL}{m_e r^3} = B_{FS}}$$

9.21) CONTINUED

c) USE $L_{2p} = 2\hbar$ (BOHR MODEL) AND $r = 4a_0$ TO FIND $B$ AND THE ENERGY DIFFERENCE BETWEEN LEVELS.

$$B_{FS} = \frac{\mu_0}{4\pi} \frac{eL}{m_e r_c^2}$$

SUBSTITUTING VALUES $\mu_0 = 4\pi \times 10^{-7}\ N/A^2$, $\hbar = 1.05 \times 10^{-34}\ J \cdot s$

$$B_{FS} = \frac{(4\pi \times 10^{-7})}{4\pi} \frac{(1.6 \times 10^{-19})[2(1.05 \times 10^{-34})]}{(9.11 \times 10^{-31})[64(5.29 \times 10^{-11})^3]}$$

$$\sim \frac{N}{A^2} \frac{C}{kg} \frac{J \cdot s}{m^2} \sim \left(\frac{kg \cdot \cancel{m}}{\cancel{s^2}} \frac{\cancel{s^2}}{C^2}\right)\left(\frac{\cancel{C}}{\cancel{kg}}\right)\left(\frac{kg \cdot \cancel{m^2}}{s^2} \cdot \frac{\cancel{s}}{\cancel{m^3}}\right) \sim \frac{kg}{C \cdot s}$$ UNITS WORK!

$$|\vec{F}| = |q\vec{v} \times \vec{B}| \Rightarrow N \sim C \cdot \frac{m}{s} \cdot T \Rightarrow T \sim \frac{N \cdot s}{C \cdot m} \sim \left(\frac{kg \cdot \cancel{m}}{s^{\cancel{2}}}\right)\left(\frac{\cancel{s}}{C \cdot \cancel{m}}\right) \sim \frac{kg}{Cs}$$

$$\boxed{B_{FS} = 0.392\ T}$$

FOR THE SEPARATION OF LEVELS,

$$\Delta E = 2\mu_B B = 2\left(5.79 \times 10^{-5} \frac{eV}{T}\right)(0.392\ T)$$

$$\boxed{\Delta E_{FS} = 4.54 \times 10^{-5}\ eV}$$ COOL!