11.6 & 15) THE POWER THAT SHOULD BE RADIATED BY THE e^- IN THE BOHR ORBIT,

$$P = \frac{2kq_0^2 a^2}{3c^3}$$

SHOULD CAUSE THE ATOM TO COLLAPSE. FIND THE TIME.

a) USING THE CHAIN RULE

$$\frac{dr}{dt} = \frac{dr}{dE}\frac{dE}{dt}$$

FOR A CIRCULAR ORBIT, $E = -\frac{1}{2}\frac{ke^2}{r}$ AND THE RADIATED POWER IS ENEGY LOST BY THE e^-, SO

$$E = -\frac{1}{2}\frac{ke^2}{r} \Rightarrow r = \frac{-ke^2}{2E} \qquad \frac{dr}{dE} = +\frac{ke^2}{2E^2}$$

SUBSTITUTING E INTO THE RESULT,

$$\frac{dr}{dE} = +\frac{ke^2}{2}\left(\frac{2r}{ke^2}\right)^2 = \frac{2r^2}{ke^2}$$

THEN

$$\frac{dE}{dt} = -\frac{2ke^2a^2}{3c^3}$$

FOR THE e^- IN ITS ORBIT

$$\frac{ke^2}{r^2} = m_e a_e \Rightarrow a_e = \frac{ke^2}{m_e r^2}$$

GIVING

TO CHANGE FROM m_e TO $m_e c^2$

$$\frac{dE}{dt} = -\frac{2ke^2}{3c^2}\left(\frac{ke^2}{m_e r^2}\right)^2\frac{c^2}{c^2}$$

$$\frac{dE}{dt} = -\frac{2(ke^2)^3 c}{3(m_e c^2)^2 r^4}$$

THUS

$$\frac{dr}{dt} = \left(\frac{2\cancel{r^2}}{\cancel{ke^2}}\right)\left(-\frac{2(ke^2)^{\cancel{3}\,2} c}{3(m_e c^2)^2 \cancel{r^4}_{\,2}}\right) = -\frac{4(ke^2)^2 c}{3(m_e c^2)^2 r^2}$$

FOR $r = a_0$

$$\boxed{\frac{dr}{dt} = -\frac{4(ke^2)^2 c}{3(m_e c^2)^2 a_0^2}}$$

THE RATE AT WHICH THE e^- FALLS IN TO THE PROTON IN H.

11.6 & 15) CONTINUED

b) ASSUME $\frac{dr}{dt}$ = CONSTANT = $-A$ FIND t TO COLLAPSE

$$\frac{dr}{dt} = -A \Rightarrow dt = -\frac{1}{A}dr$$

$$t_{COLLAPSE} = -\frac{1}{A}\int_{a_0}^{0} dr = -\frac{r}{A}\Big|_{a_0}^{0} = \frac{a_0}{A}$$

EVALUATING

$$a_0 = 5.29\times10^{-11}\,m\left(\frac{fm}{1\times10^{-15}\,m}\right)$$

$$a_0 = 5.29\times10^{4}\,fm$$

$$t_{COLLAPSE} = a_0\left(\frac{3(m_ec^2)^2a_0^2}{4(ke^2)^2c}\right)$$

$$= \frac{3}{4}\frac{(m_ec^2)^2a_0^3}{(ke^2)^2c} = \frac{3}{4}\frac{(0.511\,MeV)^2}{(1.44\,MeV\cdot fm)^2}\frac{(5.29\times10^4\,fm)^2}{(3\times10^8\,m/s^2)}(5.29\times10^{-11}\,m)$$

$$\boxed{t_{COLLAPSE} = 4.63\times10^{-11}\,s}$$ QUICK! SO WHY IS THERE STUFF?

c) MAKE A BETTE APPROXIMATION BY RECOGNIZING

$$\frac{dr}{dt} = \frac{dr}{dt} \Rightarrow dt = \frac{dr}{dr/dt} \Rightarrow t_{COLLAPSE} = \int_{a_0}^{0}\frac{dr}{dr/dt}$$

RE-WRITE dr/dt WITH r INSTEAD OF a_0

$$t_{COLLAPSE} = -\frac{3(m_ec^2)^2}{4(ke^2)^2c}\int_{a_0}^{0} r^2\,dr = -\frac{3(m_ec^2)^2}{4(ke^2)^2c}\left[\frac{r^3}{3}\Big|_{a_0}^{0}\right]$$

$$t_{COLLAPSE} = \frac{\cancel{3}(m_ec^2)^2}{4(ke^2)^2c}\frac{a_0^3}{\cancel{3}} = \frac{(m_ec^2)^2}{4(ke^2)^2c}a_0^3$$

SO THIS IS ONE THIRD OF THE PREVIOUS RESULT

$$\boxed{t_{COLLAPSE} = 1.54\times10^{-11}\,s}$$ STILL QUICK!