9.2) THE RESULT OF PROBLEM 1 CAN BE USED TO ESTIMATE THE RELATIVE PROBABILITIES OF VARIOUS VELOCITIES. PICK A SMALL INTERVAL, $\Delta v_x = 0.002 v_{x,rms}$. FOR 1 MOLE OF AN IDEAL GAS COMPUTE THE NUMBER OF MOLECULES WITHIN v_x CENTERED AT

a) $v_x = 0.01 v_{x,rms}$
b) " 0.20 "
c) " 1.00 "
d) " 5.00 "
e) " 100.0 "

FROM PROBLEM 1, THE NUMBER OF PARTICLES WITH VELOCITIES BETWEEN x AND $x + dx$ IS

$$g(v_x)\,dx = \frac{1}{\sqrt{2\pi}\, v_{x,rms}} e^{-\frac{1}{2}(v_x/v_{x,rms})^2}\,dx$$

FOR A SMALL INTERVAL, WE CAN USE A PRODUCT INSTEAD OF INTEGRATING:

$$g(v_x)\,dv_x \approx g(v_x)\Delta v_x, \quad \text{WHERE } \Delta v_x = 0.002 v_{x,rms}$$

a) FOR $v_x = 0.01 v_{x,rms}$

$$g(v_x)\,dv_x \approx \frac{1}{\sqrt{2\pi}\, v_{x,rms}} \underbrace{e^{-\frac{1}{2}\left(\frac{0.01 v_{x,rms}}{v_{x,rms}}\right)^2}}_{\approx 1} (0.002 v_{x,rms})$$

$$\approx \frac{1}{\sqrt{2\pi}}(0.99995)(0.002) \approx 7.98\times10^{-4}$$

IN A MOLE OF GAS, THERE ARE N_A MOLECULES, THUS THE NUMBER IN THIS RANGE OF SPEEDS IS

$$N_{v_x} = \left(g(v_x)\,dv_x\right)N_A = (7.98\times10^{-4})(6.022\times10^{23})$$

$$\boxed{N_{v_x} = 4.80\times10^{20} \text{ MOLECULES NEAR } 0.01 v_{x,rms}}$$

A BIG NUMBER!

b) FOR $v_x = 0.20 v_{x,rms}$

$$g(v_x)\,dv_x = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(0.2)^2}(0.002) \approx 7.82\times10^{-4}$$

$$N_{v_x} = (7.82\times10^{-4})(6.022\times10^{23}) = \boxed{4.71\times10^{20} \text{ MOLECULES NEAR } 0.20 v_{x,rms}}$$

→

9.2) CONTINUED

c) For $v_x = 1.00\, v_{x,rms}$

$$g(v_x)dv_x \approx \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(1)^2}(0.002) \approx 4.84 \times 10^{-4}$$

$$N_{v_x} = (4.84 \times 10^{-4})(6.022 \times 10^{23}) = \boxed{2.91 \times 10^{20} \text{ MOLECULES NEAR } v_{RMS}}$$

d) For $v_x = 5\, v_{x,rms}$

$$g(v_x)dv_x \approx \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(5)^2}(0.002) \approx 2.97 \times 10^{-9}$$

$$N_{v_x} = (2.97 \times 10^{-9})(6.022 \times 10^{23}) = \boxed{1.79 \times 10^{15} \text{ MOLECULES NEAR } 5v_{RMS}}$$

$10^5 = 100{,}000$ TIMES FEWER!

e) For $v_x = 100\, v_{x,rms}$

$$g(v_x)dv_x \approx \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(100)^2}(0.002) \approx 0$$

$$e^{-5000} = 3.70 \times 10^{-2172} \approx 0!$$

$$\Rightarrow g(v_x)dx = 1.07 \times 10^{-2175} \approx 0$$

$$\boxed{N_{v_x} = 0 \text{ MOLECULES NEAR } 100 v_{RMS}}$$