9.15) Using the M-B energy distribution (9.26),
a) Find the mean translational kinetic energy
b) Compare result with $\frac{1}{2}m\bar{v}^2$ and $\frac{1}{2}m\overline{v^2}$.

The M-B energy distribution is

$$F(E) = \frac{8\pi}{\sqrt{2m^3}}\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-E/kT}\sqrt{E} \qquad (9.26)$$

Find the mean energy

$$\bar{E} = \int_0^\infty \frac{8\pi}{\sqrt{2m^3}}\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-E/kT}\sqrt{E}\,E\,dE$$

Pulling out the constants,

$$\bar{E} = \frac{8\pi}{\sqrt{2m^3}}\left(\frac{m}{2\pi kT}\right)^{3/2}\int_0^\infty E^{3/2} e^{-E/kT}\,dE$$

From the CRC Tables (on TRex Appendix 7)

$$\int_0^\infty x^n e^{-ax}\,dx = \frac{\Gamma(n+1)}{a^{n+1}}$$

With $\Gamma(n+1) = n\Gamma(n)$ and $\Gamma\left(\frac{3}{2}\right) = \frac{\sqrt{\pi}}{2}$

$$\Rightarrow \int_0^\infty E^{3/2} e^{-E/kT}\,dE = \frac{\Gamma\left(\frac{5}{2}\right)}{\left(\frac{1}{kT}\right)^{5/2}} = (kT)^{5/2}\left[\left(\frac{3}{2}\right)\Gamma\left(\frac{3}{2}\right)\right]$$

$$\int_0^\infty E^{3/2} e^{-E/kT}\,dE = (kT)^{5/2}\left(\frac{3\sqrt{\pi}}{4}\right)$$

Thus

$$\bar{E} = \sqrt{\frac{\cancel{64}^{4}\pi^2}{\cancel{2m^3}}\,\frac{\cancel{m^3}}{\cancel{8}\pi^3\cancel{(kT)^3}}(kT)^{\cancel{5}\,2}\frac{9\pi}{\cancel{16}_4}} = \sqrt{\frac{9(kT)^2}{4}}$$

$$\boxed{\bar{E} = \frac{3}{2}kT}$$

¡Yes! The average energy is $\frac{3}{2}kT = \frac{1}{2}mv_{RMS}^2$!