

TRex 9.15

Find the mean translational kinetic energy using (9.26).

TRex Equation 9.26 gives the energy distribution :

$$F(E) = \frac{8\pi}{\sqrt{2m^3}} \left(\frac{m\beta}{2\pi}\right)^{3/2} \sqrt{E} e^{-\beta E} = \sqrt{\frac{4}{\pi}} \beta^{3/2} \sqrt{E} e^{-\beta E}.$$

Using x instead of E for Mathematica's convenience (it changes E to e) gives

$$\text{In}[124]:= \mathbf{F[x_]} := \sqrt{\frac{4}{\pi}} * (\beta)^{3/2} * x^{1/2} * \mathbf{Exp[-\beta*x]};$$

F[x]

$$\text{Out}[125]= \frac{2 e^{-x\beta} \sqrt{x} \beta^{3/2}}{\sqrt{\pi}}$$

Find the expectation value (weighted average) of the translational kinetic energy

$$\text{In}[132]:= \mathbf{ExpecVal[x_]} := \int_0^{\infty} x * F[x] dx$$

ExpecVal[x]

$$\text{Out}[133]= \frac{3}{2\beta} \text{ if } \text{Re}[\beta] > 0$$

Compare this to the average (non-weighted average) of the translational kinetic energy

$$\text{In}[128]:= \mathbf{AvgVal[x_]} := \frac{\int_0^{\infty} x * F[x] dx}{\int_0^{\infty} F[x] dx}$$

AvgVal[x]

$$\text{Out}[129]= \frac{3}{2\beta} \text{ if } \text{Re}[\beta] > 0$$

Oh, yeah!! $F(e)$ is normalized so

$$\text{In}[130]:= \mathbf{TotalEnergy[x_]} := \int_0^{\infty} F[x] dx$$

TotalEnergy[x]

$$\text{Out}[131]= 1 \text{ if } \text{Re}[\beta] > 0$$

Define the constants, assume $t = 20^\circ \text{C}$ in order to plot the probability density.

```
In[134]:= BoltzConst = 8.62 * 10-5;
```

```
T = 393.15;
```

```
 $\beta = 1 / (T * \text{BoltzConst});$ 
```

```
ExpecVal[x]
```

```
AvgVal[x]
```

```
TotalEnergy[x]
```

```
Out[137]= 0.0508343
```

```
Out[138]= 0.0508343
```

```
Out[139]= 1.
```

OK! All the numbers are correct, so plot $F(x)$ and F_{mp} .

```
In[140]:= pMBE = Plot[F[x], {x, 0, 0.25},
```

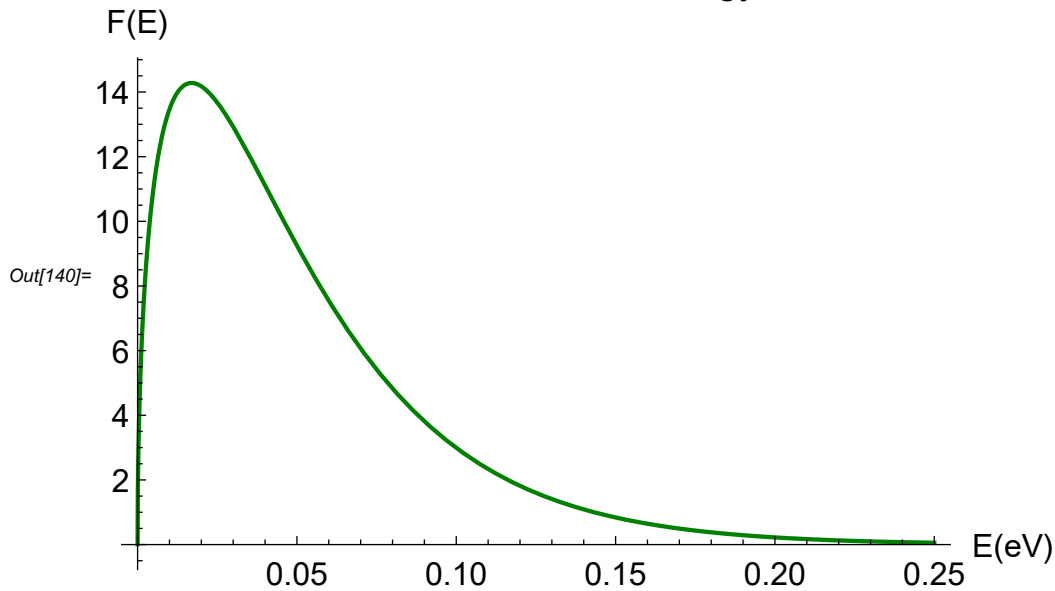
```
BaseStyle → {FontFamily → Helvetica, FontSize → 12, FontColor → RGBColor[0, 0.5, 0]},
```

```
PlotStyle → {{RGBColor[0, 0.5, 0]}},
```

```
AxesLabel → {"E(eV)", "F(E)"},
```

```
PlotLabel → "The Maxwell-Boltzmann Energy Distribution"]
```

The Maxwell-Boltzmann Energy Distribution



```
In[141]:= Export["TRex_Pr09-15.pdf", SelectedNotebook[]]
```

```
Out[22]= TRex_Pr09-15.pdf
```