9.18) MAXWELL-BOLTZMANN STATISTICS ARE VALID IF THE PARTICLE WAVE FUNCTIONS DON'T OVERLAP.

a) USING DE BROGLIE, SHOW

$$\lambda = \frac{h}{(3mkT)^{1/2}}$$

b) FOR $N/V = \frac{1}{d^3}$, SHOW THAT $\lambda \ll 1$ BECOMES

$$\left(\frac{N}{V}\right)\frac{h^3}{(3mkT)^{3/2}} \ll 1$$

a) DE BROGLIE SAID

$$\lambda = \frac{h}{p}$$

$\rightarrow E = \frac{p^2}{2m} \Rightarrow p = \sqrt{2mE}$

TAKING $p = mv$ AND $E = \frac{1}{2}mv^2 = \frac{3}{2}kT$,

$$\lambda = \frac{h}{\sqrt{2mE}} = \boxed{\frac{h}{(3mkT)^{1/2}} = \lambda}$$

WAVELENGTH OF A PARTICLE AT T

b) THE VOLUME OF A GAS IS THE NUMBER OF PARTICLES TIMES THEIR SEPARATION, THUS

$$\frac{N}{V} = \frac{1}{d^3} \Rightarrow d = \left(\frac{V}{N}\right)^{1/3}$$

REQUIRING $\lambda \ll d$ GIVES

$$\frac{h}{(3mkT)^{1/2}} \ll \left(\frac{V}{N}\right)^{1/3}$$

CUBING BOTH SIDES & REARRANGING GIVES

$$\boxed{\left(\frac{N}{V}\right)\left[\frac{h}{(3mkT)^{1/2}}\right]^3 \ll 1}$$ QED!

CONDITION FOR M-B STATS TO BE VALID

→

9.18) CONTINUED

c) DETERMINE WHETHER M-B STATS ARE VALID FOR
i) Ar GAS AT STP
ii) CONDUCTION e⁻'s IN SILVER AT T = 293 K

ci) AT STP, 1 MOLE OF IDEAL GAS HAS V = 22.4 ℓ

$$\Rightarrow \left(\frac{N}{V}\right)_{Ar,STP} = \frac{6.02\times10^{23}\ Ar}{22.4\ \ell}\left(\frac{1000\ \ell}{m^3}\right) = 2.69\times10^{25}\ \frac{Ar}{m^3}$$

TAKING (p668 TZDII)

$$m_{Ar} = 39.962\ u\left(\frac{1.66\times10^{-27}\ kg}{u}\right) = 6.63\times10^{-26}\ kg$$

$$\Rightarrow \left(\frac{N}{V}\right)\left[\frac{h}{\sqrt{3m_{Ar}kT}}\right]^3 = (2.69\times10^{25})\left[\frac{6.63\times10^{-34}}{\sqrt{3(6.63\times10^{-26})(1.38\times10^{-23})(293)}}\right]^3$$

$$= (2.69\times10^{25})(1.28\times10^{-32})$$

$$\boxed{\left(\frac{N}{V}\right)\left[\frac{h}{\sqrt{3m_{Ar}kT}}\right]^3 = 3.44\times10^{-7}} \Rightarrow \text{SINCE THIS IS} \ll 1,$$

M-B STATS VALID FOR Ar AT STP!

cii) TREX TABLE 9.3 GIVES $\left(\frac{N}{V}\right)_{Ag} = 5.86\times10^{28}\ \frac{e^-}{m^3}$

$$\Rightarrow \left(\frac{N}{V}\right)\left[\frac{h}{\sqrt{3m_e kT}}\right]^3 = (5.86\times10^{28})\left[\frac{6.63\times10^{-34}}{\sqrt{3(9.11\times10^{-31})(1.38\times10^{-23})(293)}}\right]^3$$

$$= (5.86\times10^{28})(2.51\times10^{-25})$$

$$\boxed{\left(\frac{N}{V}\right)\left[\frac{h}{\sqrt{3m_e kT}}\right]^3 = 14701 \not\ll 1!}$$

$\therefore$ SO M-B STATISTICS ARE NOT VALID FOR CONDUCTION e⁻ IN SILVER AT 293 K!