

HOMEWORK SET 4: DEGENERACY

PROBLEMS FROM TZDII¹

8.6 Let $h(x, y)$ describe a mountain as in Problem 8.5. If the same mountain is given by the contour map in Figure 8.25, give estimates for $\partial h/\partial x$ and $\partial h/\partial y$ at points P, Q, and R and the summit, S. The scale for x and y (shown by the ruled line) are given in meters.

To estimate the gradients, assume a constant gradient between contour lines so that

$$\frac{\partial h}{\partial x} \approx \frac{\Delta \text{height in } x}{\text{length in } x}$$

$$\frac{\partial h}{\partial y} \approx \frac{\Delta \text{height in } y}{\text{length in } y}$$

At R:

$$L_{R,x} = 0.5'' \frac{1000 \text{ m}}{1.0''}$$

$$L_{R,x} = 500 \text{ m}$$

$$L_{R,y} = 0 \text{ m}$$

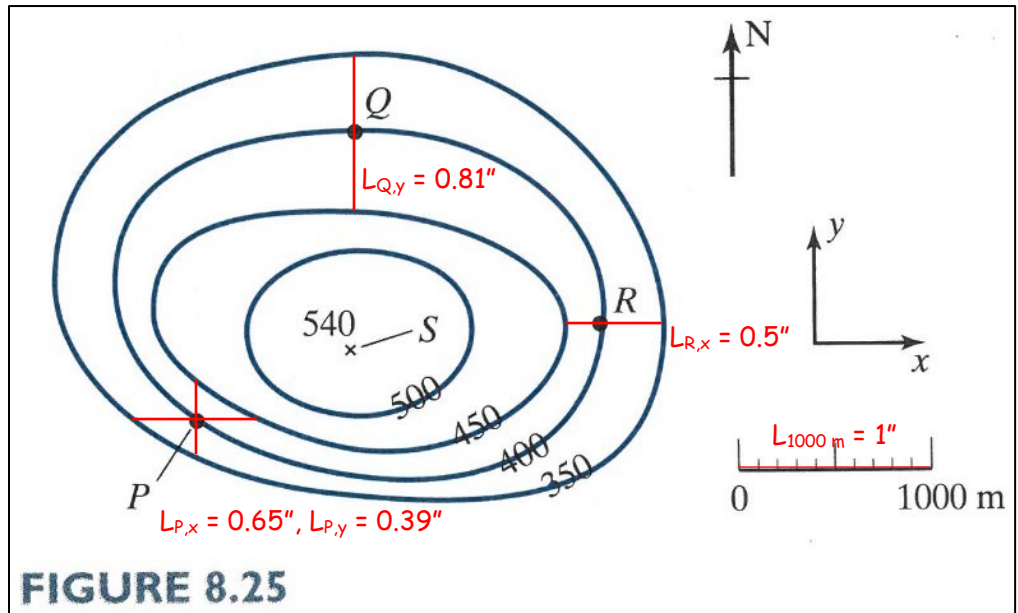


FIGURE 8.25

$$\frac{\partial h}{\partial x_R} \approx \frac{350 - 450}{500} = -0.2 = -20\% \text{ grade downhill to east} \Rightarrow \text{negative in } \hat{x}$$

$$\frac{\partial h}{\partial y_R} = 0$$

At Q:

$$L_{Q,x} = 0 \text{ m}$$

$$L_{Q,y} = 0.81'' \left(\frac{1000 \text{ m}}{1.0''} \right) = 810 \text{ m}$$

$$\frac{\partial h}{\partial x_Q} = 0$$

$$\frac{\partial h}{\partial y_Q} \approx \frac{350 - 450}{810} = -0.123 = -12\% \text{ grade downhill to north.}$$

At P:

$$L_{P,x} = 0.65'' \left(\frac{1000 \text{ m}}{1.0''} \right) = 650 \text{ m}$$

$$L_{P,y} = 0.39'' \left(\frac{1000 \text{ m}}{1.0''} \right) = 390 \text{ m}$$

$$\frac{\partial h}{\partial x_P} \approx \frac{450 - 350}{650} = 0.154 = 15\% \text{ grade uphill to east.}$$

$$\frac{\partial h}{\partial y_P} \approx \frac{450 - 350}{390} = 0.276 = 28\% \text{ grade uphill to north.}$$

At S:

Since it's the summit, it's a local maximum with the derivatives equal to zero in both x and y. There is negative slope away from S.

$$\frac{\partial h}{\partial x_S} = \frac{\partial h}{\partial y_S} = 0, \text{ but both are negative away from summit.}$$

¹ Taylor, Zafiratos, & Dubson, *Modern Physics for Scientists and Engineers*, 2nd Edition, Pearson, Prentice Hall, 2004