

1) SHOW THAT

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right) \psi = \frac{1}{r} \frac{\partial^2}{\partial r^2} (r \psi)$$

STARTING WITH THE LHS

$$\begin{aligned} \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \psi}{\partial r} \right) &= \frac{1}{r^2} \left[2r \frac{\partial \psi}{\partial r} + r^2 \frac{\partial^2 \psi}{\partial r^2} \right] \\ &= \frac{1}{r} \left[2 \frac{\partial \psi}{\partial r} + r \frac{\partial^2 \psi}{\partial r^2} \right] \end{aligned} \quad (1)$$

NOW FOR THE RHS

$$\begin{aligned} \frac{1}{r} \frac{\partial^2}{\partial r^2} (r \psi) &= \frac{1}{r} \frac{\partial}{\partial r} \left[\psi + r \frac{\partial \psi}{\partial r} \right] \\ &= \frac{1}{r} \left[\frac{\partial \psi}{\partial r} + \left(\frac{\partial \psi}{\partial r} + r \frac{\partial^2 \psi}{\partial r^2} \right) \right] \\ &= \frac{1}{r} \left[2 \frac{\partial \psi}{\partial r} + r \frac{\partial^2 \psi}{\partial r^2} \right] \end{aligned} \quad (2)$$

SINCE THE RIGHT-HAND SIDES OF (1) & (2) ARE IDENTICAL, THEIR LEFT-HAND SIDES MUST BE EQUAL, GIVING

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right) \psi = \frac{1}{r} \frac{\partial^2}{\partial r^2} (r \psi) \quad \underline{\underline{\text{QED!}}}$$

NOTE BELOW: TOWNSEND SUBSTITUTES THE RHS OF THIS FOR THE LHS IN WRITING (6.63) AFTER (6.62)