8.30 ALTERED a) WRITE DOWN THE Θ EQUATION FOR $\ell = 2$, $m = 1$. b) THAT $\Theta_{2,1}(\theta)$ IS A SOLUTION

a) THE GENERAL Θ EQUATION IS (8.65)

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left[\sin\theta\frac{d\Theta}{d\theta}\right] + \left[\ell(\ell+1) - \frac{m^2}{\sin^2\theta}\right]\Theta = 0 \qquad (8.65)$$

FOR $\ell = 2$, $m = 1$ THIS BECOMES

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left[\sin\theta\frac{d\Theta}{d\theta}\right] + \left[6 - \frac{1}{\sin^2\theta}\right]\Theta = 0 \qquad (2)$$

b) FROM TABLE 8.1

$$\Theta_{2,1}(\theta) = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta = -A\sin\theta\cos\theta$$

TAKE DERIVATIVE

$$\frac{d}{d\theta}\left(-A\sin\theta\cos\theta\right) = -A\left(\cos^2\theta - \sin^2\theta\right) = +A\left(\sin^2\theta - \cos^2\theta\right)$$

$$\frac{d^2\Theta}{d\theta^2} = A\frac{d}{d\theta}\left(\sin^2\theta - \cos^2\theta\right)$$

$$= A\left(2\sin\theta\cos\theta + 2\cos\theta\sin\theta\right) = 4A\sin\theta\cos\theta$$

SUBSTITUTE INTO THE DE (2)

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left[\sin\theta(A)(\sin^2\theta - \cos^2\theta)\right] - \left[6 - \frac{1}{\sin^2\theta}\right]A\sin\theta\cos\theta = 0$$

$$\frac{A}{\sin\theta}\frac{d}{d\theta}\left[\sin^3\theta - \sin\theta\cos^2\theta\right] - A\left[6 - \frac{1}{\sin^2\theta}\right]\sin\theta\cos\theta = 0$$

$$\frac{1}{\sin\theta}\left[3\sin^2\theta\cos\theta - \cos^3\theta + 2\sin^2\theta\cos\theta\right] - \quad = 0$$

$$3\sin\theta\cos\theta - \frac{\cos^3\theta}{\sin\theta} + 2\sin\theta\cos\theta - 6\sin\theta\cos\theta + \frac{\cos\theta}{\sin\theta} = 0$$

$$-\sin^2\theta\cos\theta - \cos^3\theta + \cos\theta = 0$$

$$-\sin^2\theta - \cos^2\theta + 1 = 0$$

$$-1 + 1 = 0 \quad \text{QED!} \quad \Theta_{2,1}(\theta) \text{ IS A SOLUTION}$$